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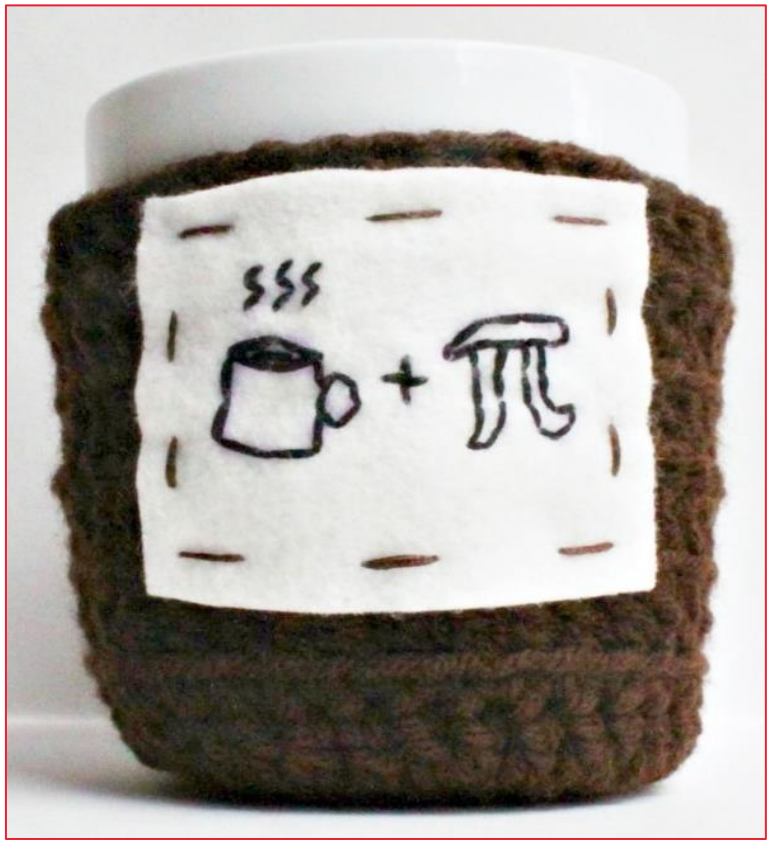

Coffee and Pi

Subtle changes to A level Pure Content 2017

Managed by MEI
Innovators in Mathematics Education

Continuing Professional Development Standard

National Centre for Excellence in the Teaching of Mathematics

be able to solve quadratic equations in a function of the unknown, where the function may be, for example, trigonometric or exponential.

Suggested resources

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SAM question (Edexcel AS SAM Paper 1):

12. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$
$$2^{2x} + 2^4 - 9(2^x) = 0$$
$$\text{Let} \quad 2^x = y$$
$$y^2 - 9y + 8 = 0$$
$$(y-8)(y-1) = 0$$
$$y = 8 \text{ or } y = 1$$
$$\text{So } x = 3 \text{ or } x = 0$$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

Express solutions through correct use of 'and' and 'or', or through set notation. Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically

Suggested resources

- http://www.teaching.martahidegkuti.com/shared/lnotes/4_collegealgebra/inequalities/inequality3.pdf
- https://undergroundmathematics.org/quadratics/inequalities-for-some-occasions

Express solutions through correct use of 'and' and 'or', or through set notation. Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically

SAM question (OCR A level SAM Pure & Mech):

1 **(i)** If $|x| = 3$, find the possible values of $|2x - 1|$. **[3]**

(ii) Find the set of values of x for which $|2x - 1| > x + 1$. Give your answer in set notation. **[4]**

Use of functions in modelling, including consideration of limitations and refinements of the models.
• suggest how a model could be improved.
• give suggestions as to when a particular model might break down or why it is only appropriate over a particular range of values.

Suggested resources

- https://undergroundmathematics.org/calculus-meets-functions/keep-your-distance
- https://undergroundmathematics.org/thinking-about-functions/picture-the-process-i

Use of functions in modelling, including consideration of limitations and refinements of the models.

- suggest how a model could be improved.
- give suggestions as to when a particular model might break down or why it is only appropriate over a particular range of values.

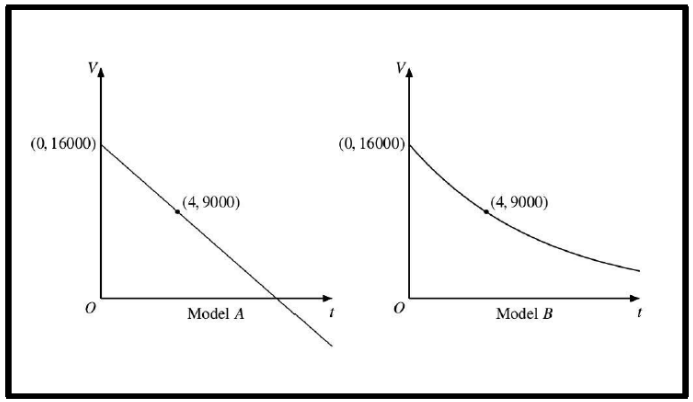
SAM question (Edexcel A level SAM Paper 1):

6. A car dealer wishes to model the value of a certain type of car.

The initial price of the car is £16 000 and the value after 4 years is expected to be £9 000.

In a simple model, the value of the car, £V, depends only on the age of the car, t years.

The diagram below shows the graphs of two possible models over 12 years.



(a) Explain why Model A is an unrealistic model for cars over 10 years of age.

(1)

It is given that the equation for model B is of the form $V = pe^{kt}$, where p and k are constants.

(b) Find the equation for model B.

(3)

Saima wants to know the value of her car when it is 3 years old.

(c) (i) Use model A to predict the value of Saima's car.

(ii) Use model B to predict the value of Saima's car.

(2)

(d) Write down one possible refinement of either model A or model B.

(1)

Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; use the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.

Suggested resources

- https://undergroundmathematics.org/circles/finding-circles
- https://undergroundmathematics.org/circles/belt

Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; use the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.

SAM question (AQA A level SAM Paper 1):

11 A circle with centre C has equation $x^2 + y^2 + 8x - 12y = 12$

11 (a) Find the coordinates of C and the radius of the circle.

[3 marks]

11 (b) The points P and Q lie on the circle.

The origin is the midpoint of the chord PQ.

Show that PQ has length $n\sqrt{3}$, where n is an integer.

[5 marks]

Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$ and $\tan\theta \approx \theta$ where θ is in radians.

Suggested resources

- https://www.tarquingroup.com/media/wysiwyg/Sample_sections_-_teacher.pdf
- https://www.youtube.com/watch?v=-IRHJmBPWw4

Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$ and $\tan\theta \approx \theta$ where θ is in radians.

SAM question (AQA A level SAM Paper 1):

3 When θ is small, find an approximation for $\cos 3\theta + \theta \sin 2\theta$, giving your answer in the form $a + b\theta^2$

[3 marks]

Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.

Suggested resources

- https://undergroundmathematics.org/exp-and-log/plotting-planets

- See also old MEI C3 papers

Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.

SAM question (Edexcel AS level SAM Paper 1):

14.

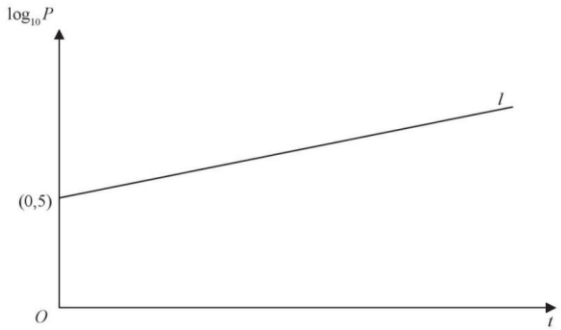


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

differentiation from first principles for small positive integer powers of x

Suggested resources

- https://www.tes.com/teaching-resource/differentiation-from-first-principles-6146715
- https://undergroundmathematics.org/introducing-calculus/zooming-in

differentiation from first principles for small positive integer powers of x

SAM question (OCR AS level SAM Pure & Mech):

7 Differentiate $f(x) = x^4$ from first principles. **[5]**

Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.

Suggested resources

- https://undergroundmathematics.org/calculus-meets-functions/gradients-of-gradients

Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.

SAM question (AQA Teaching Guidance):

1 A curve is given by the equation $f(x) = x^3 - 3x^2 + 1$

Find the range of values of x for which the curve is concave.

Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams. Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$. Understand how such methods can fail.

Suggested resources

- https://undergroundmathematics.org/calculus-of-powers/r8231
- https://www.tes.com/teaching-resource/numerical-methods-6148274
- https://www.tes.com/teaching-resource/iteration-and-graphical-iteration-staircase-and-cobweb-11625924

Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams. Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$. Understand how such methods can fail.

SAM question (AQA Teaching Guidance):

1 (a) The equation $e^{-x} - 2\sqrt{x} = 0$ has a single root, α .

Show that α lies between 3 and 4.

(b) Use the recurrence relation

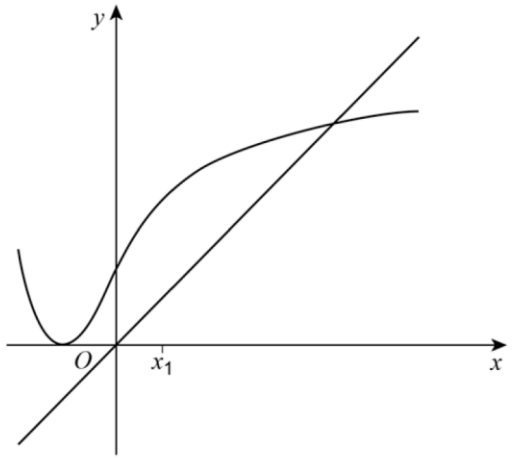
$$x_{n+1} = \left(2 - e^{-x_n}\right)^2$$

With

$$x_1 = 3.5$$

to find x_2 and x_3 giving your answers to three decimal places.

(c) The diagram shows parts of the graphs of $y = \left(2 - e^{-x}\right)^2$ and $y = x$, and a position of x_1.



On the diagram, draw a staircase or cobweb diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.

Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.

Suggested resources

Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.

SAM question (AQA A level SAM Paper 1):

See also

http://www.resourceaholic.com/p/new-level-support.html

The Further Mathematics Support Programme

Our aim is to increase the uptake of AS and A level Further Mathematics to ensure that more students reach their potential in mathematics.

The FMSP works closely with school/college maths departments to provide professional development opportunities for teachers and maths promotion events for students.

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