FMSP | Let maths take you further

The Further Mathematics Support Programme

Our aim is to increase the uptake of AS and A level Further Mathematics to ensure that more students reach their potential in mathematics.

The FMSP works closely with school/college maths departments to provide professional development opportunities for teachers and maths promotion events for students.

To find out more please visit *www.furthermaths.org.uk*

A level Mathematics: what's new in Statistics

Outline of meeting

1600	Introduction – what's new
16010	Significance and hypothesis testing with the binomial distribution
1645	*Tea*
1705	Normal distribution and hypothesis testing
1740	Large data sets, sampling and accredited questions
1800	Close

Introduction

Aims:

- To meet the major new topics at AS and A level Mathematics
- To understand how teaching may need to change to met the change in examining ethos
- To try some activities which can be used in the classroom


BBC FOUR

The Joy of Stats - Hans Rosling

- How could this video change the way people look at statistics?
    - this is part of an hour long program available on YouTube; other presentations are on TED – worth a look!

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BIG data

https://vimeo.com/121940231

Content - General

- The new content is designed to build on the new GCSE content
- Content is 100% specified for Mathematics and 50% specified for Further Mathematics
- **Mechanics** and Statistics element is prescribed at both AS and full A level

What's new?

- Large data sets
- Hypothesis testing at AS for all awarding bodies

- A new ethos around what 'statistics' means at A level
  - ~~Calculations~~
  - Use of technology
  - Interpretation
- No tables – use calculators instead

Statistics Content

L Data presentation and interpretation

	Content
L1	[Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency] [Connect to probability distributions]
L2	[Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded)] [Understand informal interpretation of correlation] [Understand that correlation does not imply causation]

Statistics Content

L3	**[Interpret measures of central tendency and variation, extending to standard deviation]** **[Be able to calculate standard deviation, including from summary statistics]**
L4	**[Recognise and interpret possible outliers in data sets and statistical diagrams]** **[Select or critique data presentation techniques in the context of a statistical problem]** **[Be able to clean data, including dealing with missing data, errors and outliers]**

Statistics Content

M Probability

	Content
M1	**[Understand and use mutually exclusive and independent events when calculating probabilities]** **[Link to discrete and continuous distributions]**
M2	Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables Understand and use the conditional probability formula $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
M3	Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions

Statistics Content

N Statistical distributions

	Content
N1	**[Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution]**
N2	Understand and use the Normal distribution as a model; find probabilities using the Normal distribution Link to histograms, mean, standard deviation, points of inflection and the binomial distribution
N3	Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate

Statistics Content

O Statistical hypothesis testing

	Content
O1	**[Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value]**; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded)
O2	**[Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context]** **[Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis]**
O3	Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance and interpret the results in context

Comparison with S1 content

S1 topics (current specifications)	Edexcel	AQA	OCR	MEI
Representation of Data in graphical form	✓	✓	✓	✓
Averages and Spread; Outliers and Skewness	✓	✓	✓	✓
The laws of Probability; conditional probability	✓	✓	✓	✓
Discrete Random Variables	✓	✓	✓	✓
The Binomial Distribution	✗	✓	✓	✓
Correlation and Regression	✓*	✓	✓	✗
The Normal Distribution	✓	✓	✗	✗
Sample means; Central Limit theorem	✗	✓	✗	✓
Hypothesis testing for the Binomial Distribution	✗	✗	✗	✓

Comparison with S1 content

Statistics topics in new AS level	Edexcel	AQA	OCR	MEI
Representation of Data in graphical form	S1	S1	S1	S1
Averages and Spread; outliers and Skewness	S1	S1	S1	S1
Probability; independence of events	S1	S1	S1	S1
Correlation and Regression	S1	S1	S1	S1
Discrete Random Variables	S1	S1	S1	S1
The Binomial Distribution	S2	S1	S1	S1
Hypothesis testing for the Binomial Distribution	S2	S2	S2	S1
Methods of sampling	S3	S2	S2	S2
Using large data sets; cleaning data	-	-	-	-

Comparison with S1 content

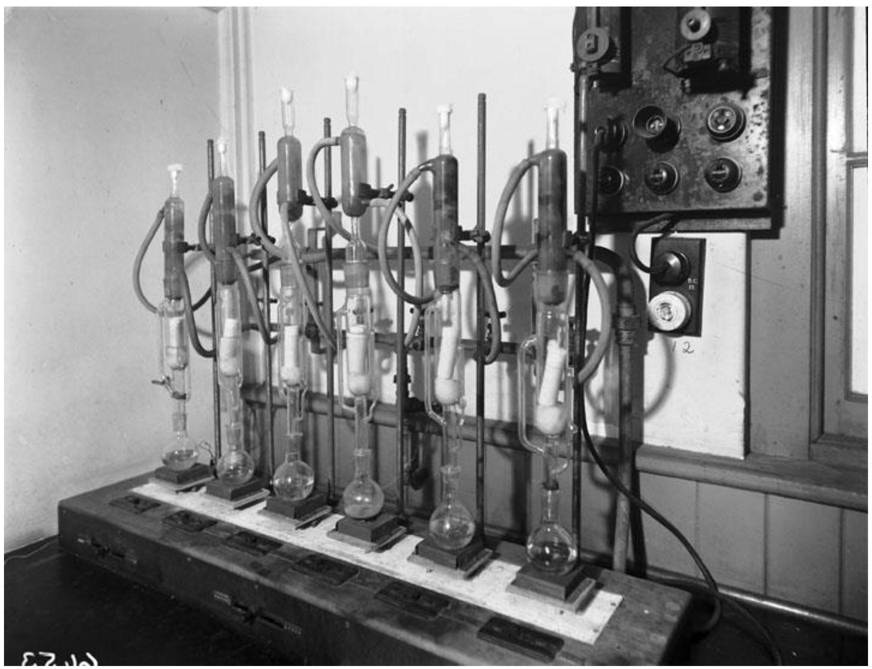
Additional Statistics topics in full A level	Edexcel	AQA	OCR	MEI
Conditional probability; Venn diagrams	S1	S1	S1	S1
Probability models for data	-	-	-	-
The Normal Distribution	S1	S1	S2	S2
Hypothesis test for the mean of a Normal Distribution	S3	S2	S2	S2
Hypothesis tests for correlation coefficient	S3	S3	S2	S2

A level Mathematics: significance

Aims:

- Appreciate that statistics is not always right or wrong
- Decide whether a result is statistically significant
- appreciate how differences in inference can occur

Significance



"The mean time without the catalyst was 28.1 s.

The mean time with the catalyst was 27.3 s.

So I conclude that the catalyst speeds up the reaction."

Catalyst data

Scenario A

Times without catalyst

28.0 28.1 28.1 28.2

Mean = 28.1 s

Times with catalyst

27.1 27.2 27.4 27.5

Mean = 27.3 s

Scenario B

Times without catalyst

26.9 27.4 28.5 29.6

Mean = 28.1 s

Times with catalyst

26.9 26.9 27.0 28.4

Mean = 27.3 s

Being certain

In *maths* you can be sure that some things are always true; you can prove them.

'If you add two odd numbers you always get an even number.'

In *statistics* you cannot be so certain. You collect evidence from a sample, and you come to a conclusion about the population.

'There is good evidence to suggest that the catalyst speeds up the reaction.'

'There is not enough evidence to suggest that coffee affects coordination.'

Experimenter bias

There is evidence to suggest association between playing violent video games and real world behaviour

There is not enough evidence to suggest association between playing violent video games and real world behaviour

A level Mathematics: hypothesis testing with the binomial distribution

Aims:

- Calculate binomial distributions
- Use the binomial distribution as a model
- Set up a hypothesis test using the binomial distribution
- Solve problems that relate to the binomial distribution

Quiz 1

1. How long did the hundred years war last?

A. 92 years B. 100 years C. 108 years D. 116 years

2. Which country makes Panama hats?

A. Costa Rica B. Ecuador C. Nicaragua D. Panama

3. From which animal do we get catgut?

A. Cats B. Horses C. Sheep D. Sheep and Horses

4. In which month do Russians celebrate the October Revolution?

A. September B. October C. November D. December

5. What is a camel's hair brush made of?

A. Camel hair B. Dog hair C. Monkey fur D. Squirrel fur

Quiz 1 contd.

6. What animal are the Canary Islands in the Pacific named after?

A. Canaries B. Dogs C. Cats D. Parrots

7. What was King George VI's first name?

A. Albert B. Edward C. George D. Philip

8. What colour is a purple finch?

A. Crimson B. Green C. Purple D. Yellow

9. Where are Chinese gooseberries from?

A. Australia B. China C. India D. New Zealand

10. What is the colour of the black box in a commercial aeroplane?

A. Black B. Brown C. Grey D. Orange

Quiz 2

Write down the answers to the following multiple choice Quiz 2

1.	A.	B.	C.	D.
2.	A.	B.	C.	D.
3.	A.	B.	C.	D.
4.	A.	B.	C.	D.
5.	A.	B.	C.	D.
6.	A.	B.	C.	D.
7.	A.	B.	C.	D.
8.	A.	B.	C.	D.
9.	A.	B.	C.	D.
10.	A.	B.	C.	D.

Quiz answers – same for both quizzes

Correct answers are:

1. D 2. B 3. D 4. C 5. D

6. B 7. A 8. A 9. D 10. D

- Count the number of correct answers, r.
- What do you think the two distributions will look like? Why?

Guessing or not?

- How many questions does someone have to get correct before we can conclude that they are not guessing?
- Somebody gets 5 questions correct. What evidence do we have to suggest that they are not just guessing?

We can use a hypothesis test to make a decision

What is a hypothesis test?

- Let's **assume** they are guessing.

- Then we have a probability to work with:
  P(correct) = ¼

- We find the probability of getting 5 questions correct – or an even better result.

- And then decide whether our **assumption** was reasonable or not.

What is a hypothesis test?

Start with a **null hypothesis**:

EITHER

Use this to calculate the probability of observed data (or more extreme values).
This is called the ***p-value***.
Reject the null hypothesis if it is very unlikely.

OR

Find the values which would be very unlikely if the null hypothesis were true.
This is called the ***critical region***.

What is meant by "very unlikely"?

The measure of "very unlikely" needs to be chosen before carrying out the hypothesis test.

This is to prevent biased decisions.

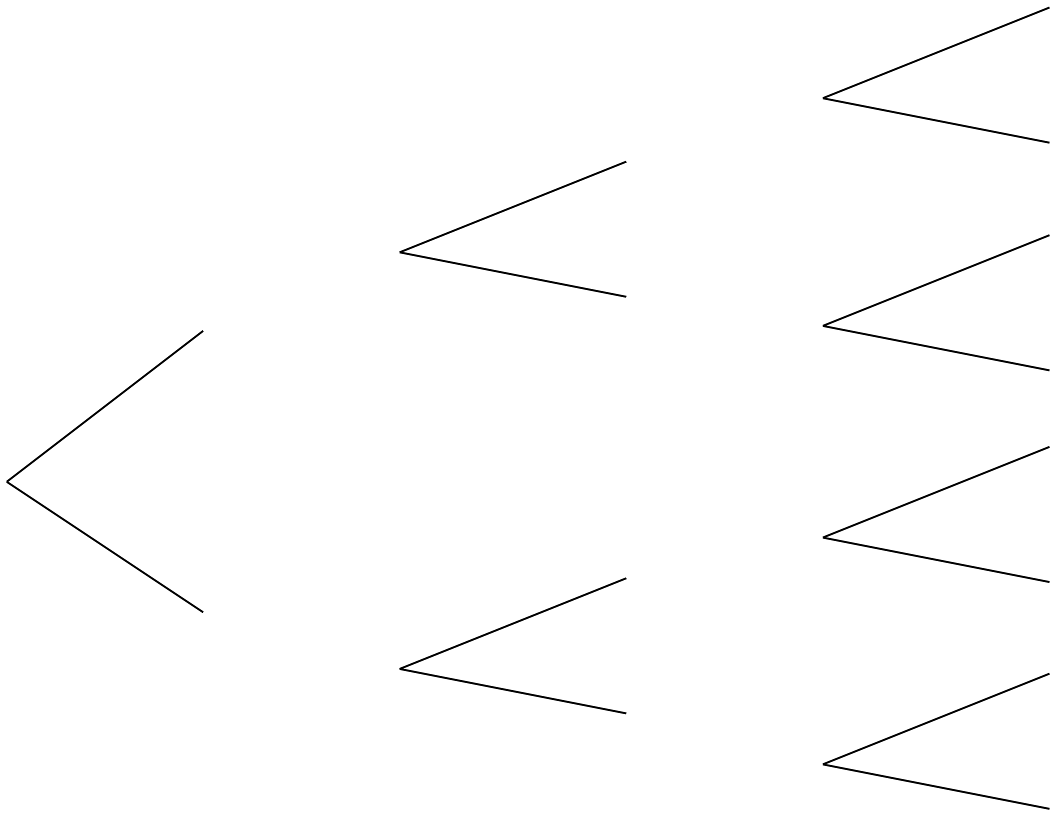
The **significance level** tells us what we mean by "very unlikely" – it is often 5% but other values (e.g. 1%) are used.

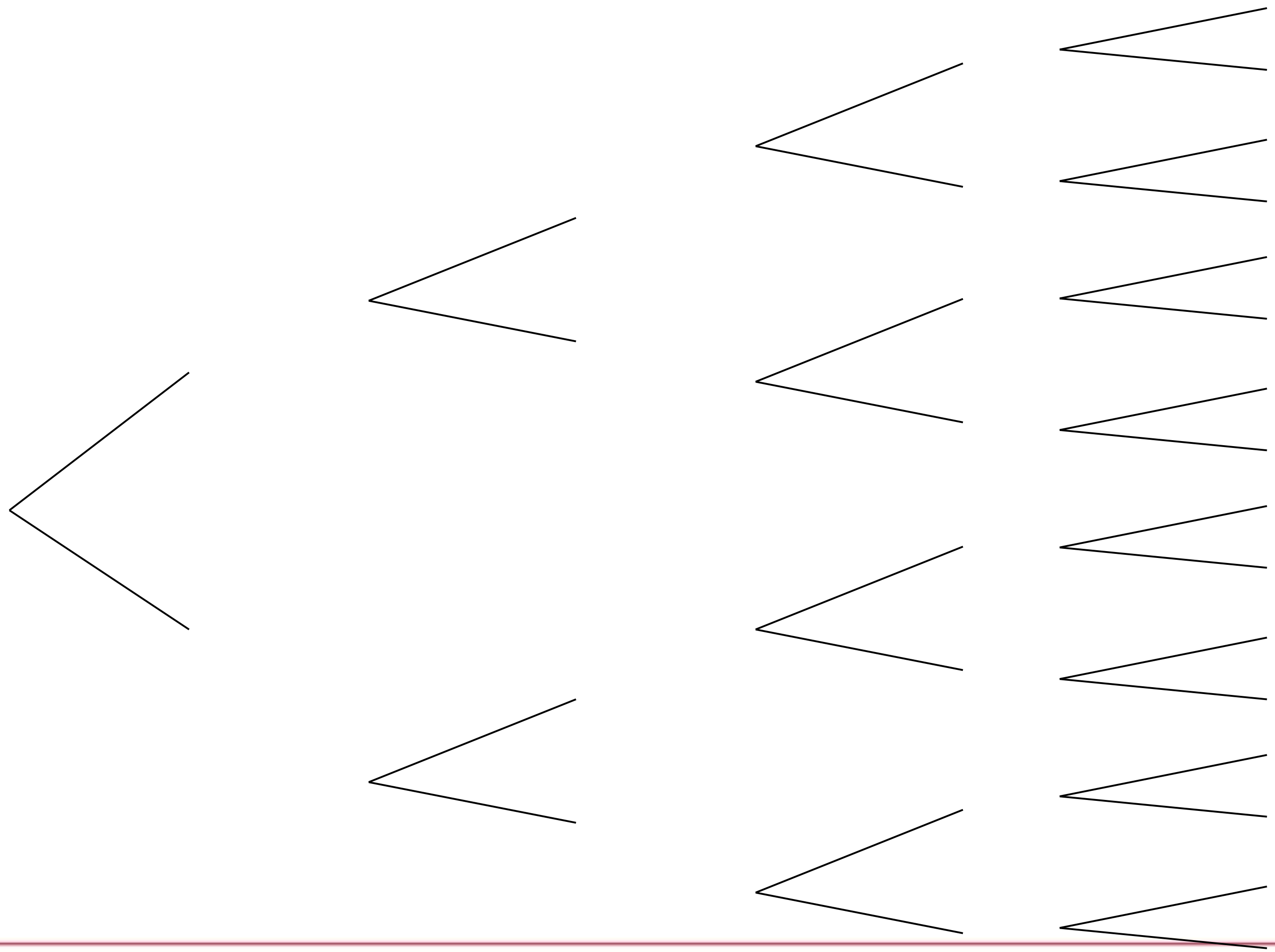
Calculating the probability

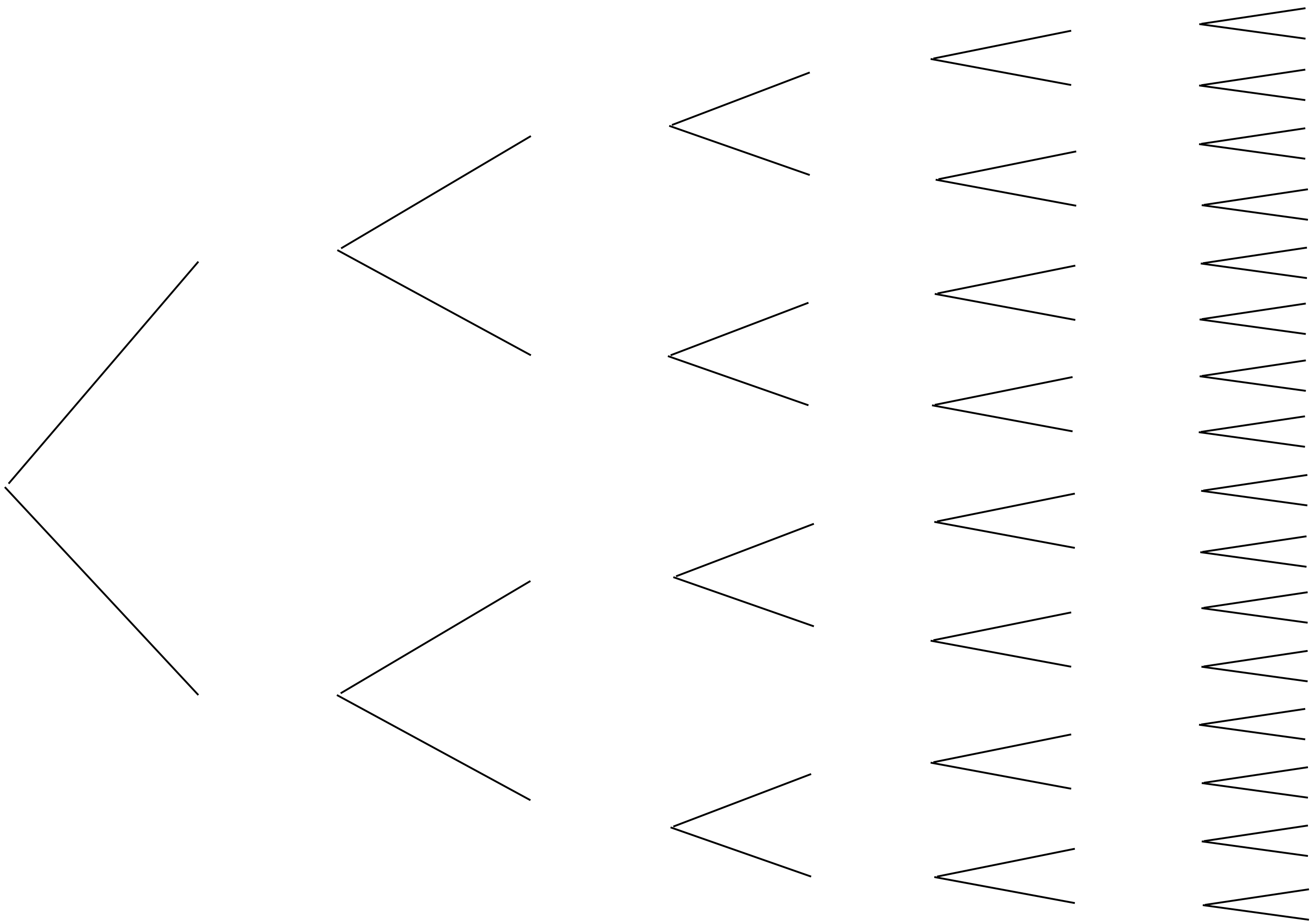
- Now we have to work out the probability of getting 5 or more questions correct.
- You could use this context to introduce the Binomial Distribution to your students, probably immediately after teaching them about Binomial Expansions in Pure, and returning to the hypothesis once they have got to grips with the probabilty.

Aside…

- Very quickly, some ideas for introducing the Binomial Distribution, for those who haven't taught it before…







Binomial probabilities

Number of successes

Number of failures

$$P(X = r) = {}^{n}C_{r} \times p^{r} \times q^{n-r}$$

Number of routes through the tree diagram

Probability of success

Probability of failure

What is the question?

- Here is a probability.
- Write a question to which this is the answer.

$^{5}C_{3} \times 0.2^{3} \times 0.8^{2}$

$^{6}C_{4} \times 0.9^{4} \times 0.1^{2}$

$^{4}C_{2} \times 0.1^{2} \times 0.9^{2}$

$^{6}C_{5} \times 0.7^{5} \times 0.3^{1}$

$^{5}C_{2} \times 0.8^{2} \times 0.2^{3}$

$${}^{5}C_{0} \times 0.9^{5} + {}^{5}C_{1} \times 0.1 \times 0.9^{4}$$

$${}^{5}C_{5} \times 0.1^{5} + {}^{5}C_{4} \times 0.1^{4} \times 0.9$$

$${}^{6}C_{6} \times 0.6^{6} + {}^{6}C_{5} \times 0.6^{5} \times 0.4^{1} + {}^{6}C_{4} \times 0.6^{4} \times 0.4^{2}$$

$${}^{10}C_{0} \times 0.1^{10} + {}^{10}C_{1} \times 0.9 \times 0.1^{9} + {}^{10}C_{2} \times 0.9^{2} \times 0.1^{8}$$

$${}^{8}C_{8} \times 0.4^{8} + {}^{8}C_{7} \times 0.4^{7} \times 0.6$$

Language Issues

Matching Activity

Scenario:

The probability of winning a game is always 0.6, and there are 8 games left to play.

Match any statements that describe the same situation

P(win fewer than 3)	P(win no more than 3)
P(win at least 3)	P(lose more than 5)
P(win exactly 3)	P(win more than 3)

$.6^3+{}^8C_4 0.4^4\times0.6^4+{}^8C_3 0.4^3\times0.6^5+{}^8C_2 0.4^2\times0.6^6+{}^8C_1 0.4\times0.6^7+0.6^8$
${}^8C_2 0.6^2\times0.4^6+{}^8C_1 0.6\times0.4^7+0.4^8$
${}^8C_6 0.4^6\times0.6^2+{}^8C_7 0.4^7\times0.6+0.4^8$

10 questions

P(10 correct) =
P(9 correct) =
P(8 correct) =
P(7 correct) =
P(6 correct) =
P(5 correct) =
P(4 correct) =
P(3 correct) =
P(2 correct) =
P(1 correct) =
P(0 correct) =

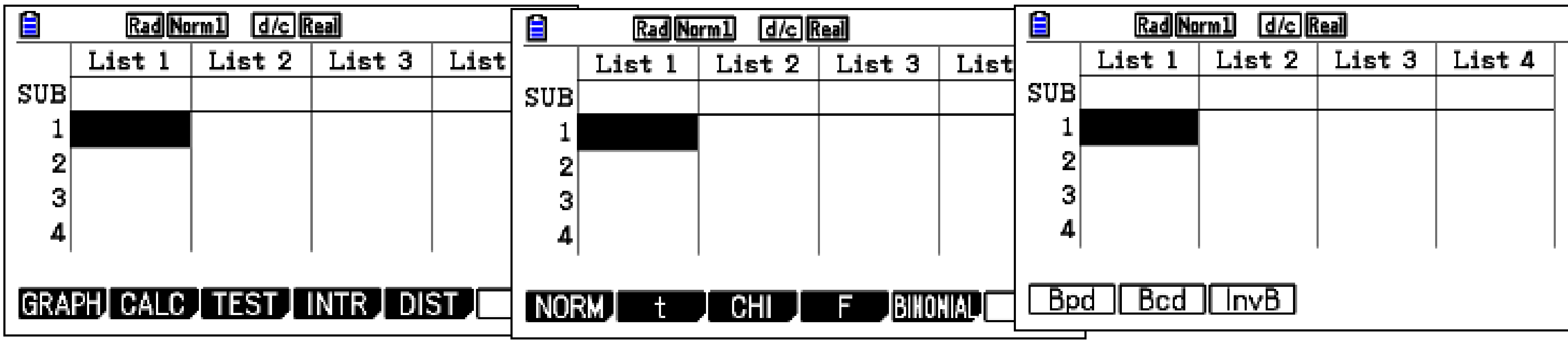
How many questions does someone have to get correct before we decide they are **not** guessing?

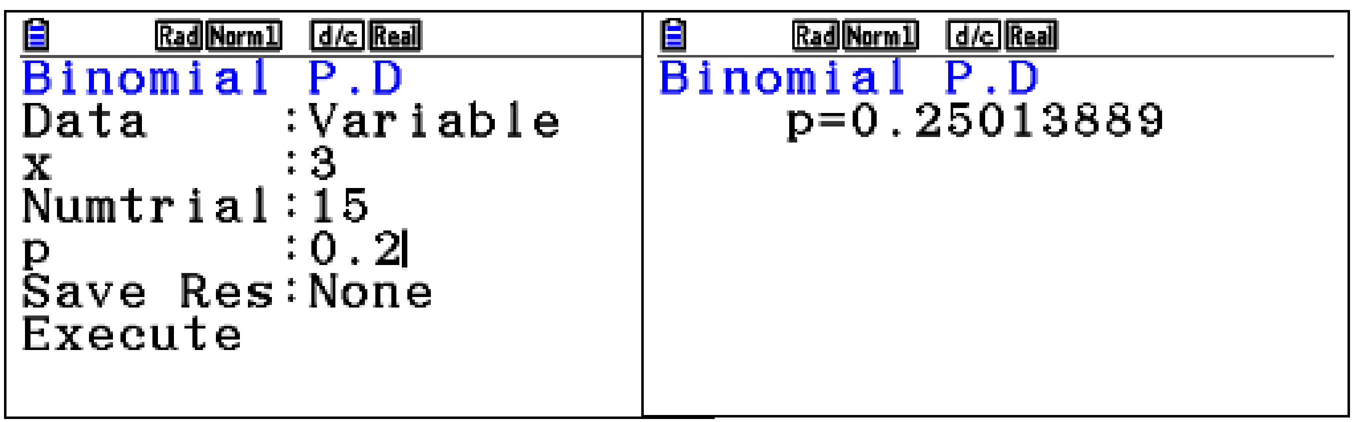
Using a graphical calculator

- Using a graphical calculator, work out the probability of getting each possible score from a quiz with 10 questions

Using a calculator

(a) Probabilities of type P($X = x$): Bpd eg B(15, 0.2) P(X = 3)





10 questions

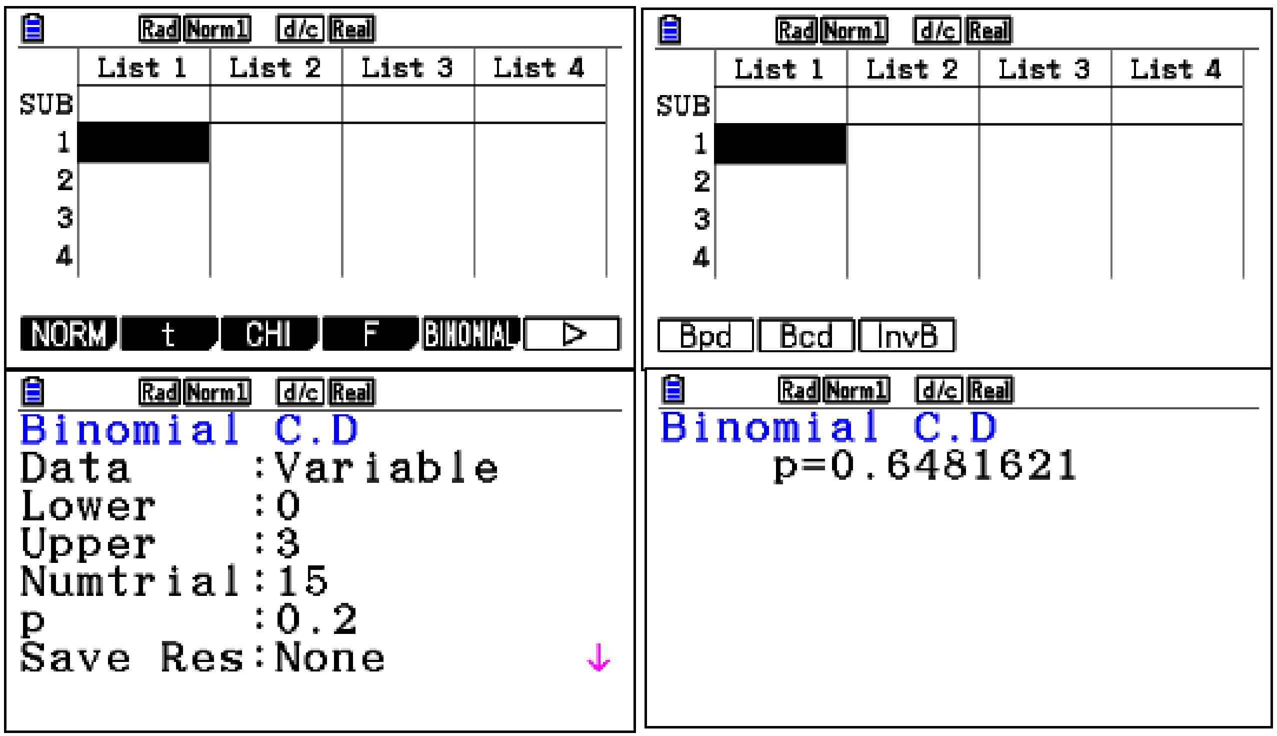
P(10 correct) = 0.00000
P(9 correct) = 0.00003
P(8 correct) = 0.00039
P(7 correct) = 0.00309
P(6 correct) = 0.01622
P(5 correct) = 0.05840
P(4 correct) = 0.14599
P(3 correct) = 0.25028
P(2 correct) = 0.28156
P(1 correct) = 0.18771
P(0 correct) = 0.05631

How many questions does someone have to get correct before we decide they are **not** guessing?

Cumulative probabilities

(b) Probabilities of type $P(X \le x)$: Bcd

eg B(15, 0.2) $P(X \le 3)$



10 questions

P(10 correct) = 0.00000

P(9 or more correct) = 0.000003

P(8 or more correct) = 0.000416

P(7 or morecorrect) = 0.003506

P(6 or more correct) = 0.019727

P(5 or more correct) = 0.078126

P(4 or more correct) = 0.22412

P(3 or more correct) = 0.4744

P(2 or more correct) = 0.75597

P(1 or more correct) = 0.94368

P(0 or more correct) = 1

Formalising the hypothesis test

- Define p
- State the null hypothesis and alternative hypothesis
- Describe the distribution if the null hypothesis is true
- Decide if it is a one-tailed (only large or only small values matter) or two-tailed test (both large and small values matter)
- Find the cumulative probability of the observed value of X and the values more extreme – *or find the critical region*
- Compare the probability and significance level
- **Decide whether to accept or reject the null hypothesis**
- **State the decision in a way that relates to the original situation**

Say what p stands for	
State the null hypothesis; (they are guessing)	
State the alternative hypothesis (they know the answer)	
Decide what the distribution seen in the sample would be if the null hypothesis is true	
Decide whether large or small values of X (or both) would lead to rejection of null hypothesis	
Find the critical region for a 5% significance level	
Compare observed value with the critical region	
Decide whether to accept or reject the null hypo	
State the decision in a way that relates to the original situation	

Say what *p* stands for	*p is the probability of getting a question correct*
State the null hypothesis; (they are guessing)	H_0: *p = 0.25* as there are four different options
State the alternative hypothesis (they know the answer)	H_1: *p > 0.25*
Decide what the distribution seen in the sample would be if the null hypothesis is true	X$\sim B(10,0.25)$ where *X* is the number of questions answered correctly
Decide whether large or small values of *X* (or both) would lead to rejection of null hypothesis	Large values of *X* would lead to rejection of H_0
Find the critical region for a 5% significance level	$P(X \geq 5) = 0.07813$ $P(X \geq 6) = 0.01973$ Critical regions is X ≥ 6
Compare observed value with the critical region	5 is not in the critical region
Decide whether to accept or reject the null hypo	X= 5 is **not** in tail of distribution so there is NO EVIDENCE TO REJECT at 5% significance level
State the decision in a way that relates to the original situation	There is insufficient evidence to suggest this person is not just guessing answers.

Other practical hypothesis tests you could try…

- Real-life situations will always be more motivating
- Collecting data is time-consuming but enables fruitful discussion about sampling, bias etc.

The Taste Test

- Striders think their crisps taste better than supermarket brands.
- They've done research that says that 7 out of 10 people prefer Striders crisps.
- They use this in their advertising.
- Staylily supermarket think they're wrong. They want to challenge the advertisement.
- Staylily think that fewer than 7 out of 10 people prefer Striders crisps.

Telepathy

- Set up an experiment to test if pairs of students can communicate which playing card they are looking at.

Use other subject areas

- Get students to make and test hypotheses relating to the other subjects that they study (Biology, Psychology, Geography…).

Two tailed tests – using critical regions

The probability that a certain type of seed germinates is estimated to be 0.65. A new method of storing the seeds is being trialled, and the company wish to know whether this changes the proportion of seeds that germinate.
A sample of 40 seeds is tested and 19 of the seeds germinate. Is there evidence at the 5% level of a change in the proportion of seeds that germinate?

Two tailed tests – using critical regions

The probability that a certain type of seed germinates is estimated to be 0.65. A new method of storing the seeds is being trialled, and the company wish to know whether this changes the proportion of seeds that germinate.
A sample of 40 seeds is tested and 19 of the seeds germinate. Is there evidence at the 5% level of a change in the proportion of seeds that germinate?

As we are asked about a change of proportion, both larger and smaller numbers of seeds would matter so it is a two-tailed test

Critical Regions

- We can work out a critical region

$$X \sim B(40, 0.65)$$

Critical Regions

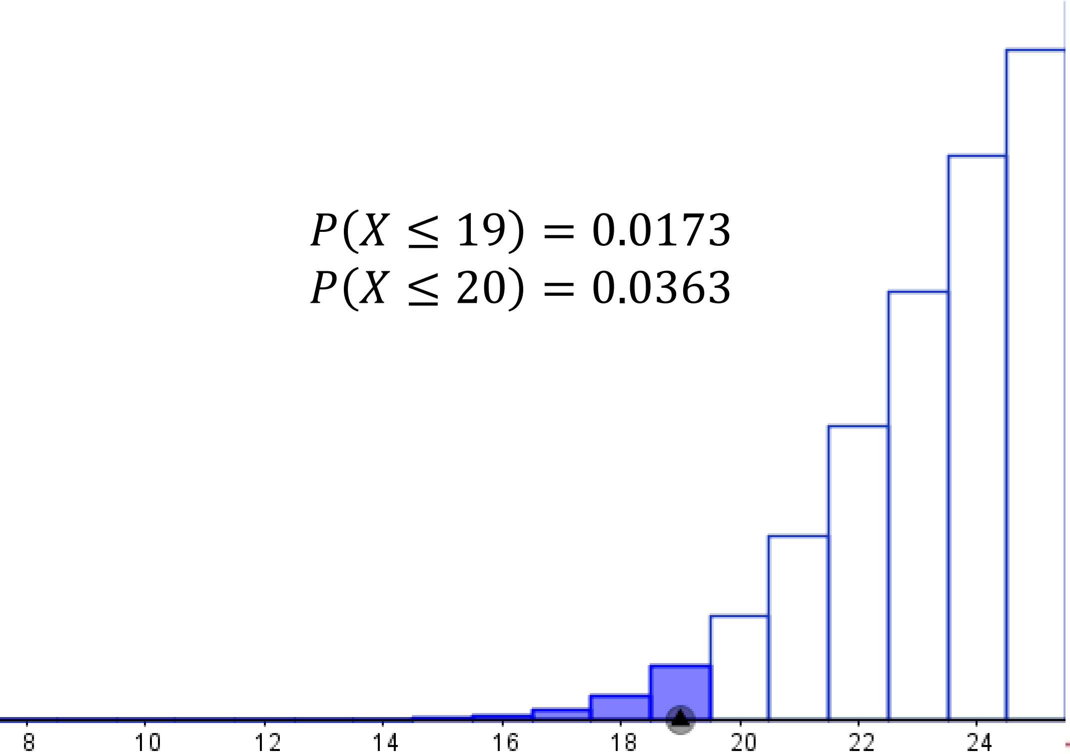
- We can work out a critical region

$$X \sim B(40, 0.65)$$

- As it is a two-tailed test at 5% significance, we need 2.5% at either end of the scale

Critical Regions

- $X \sim$ B(40, 0.65)
- 2.5% at either end of the scale



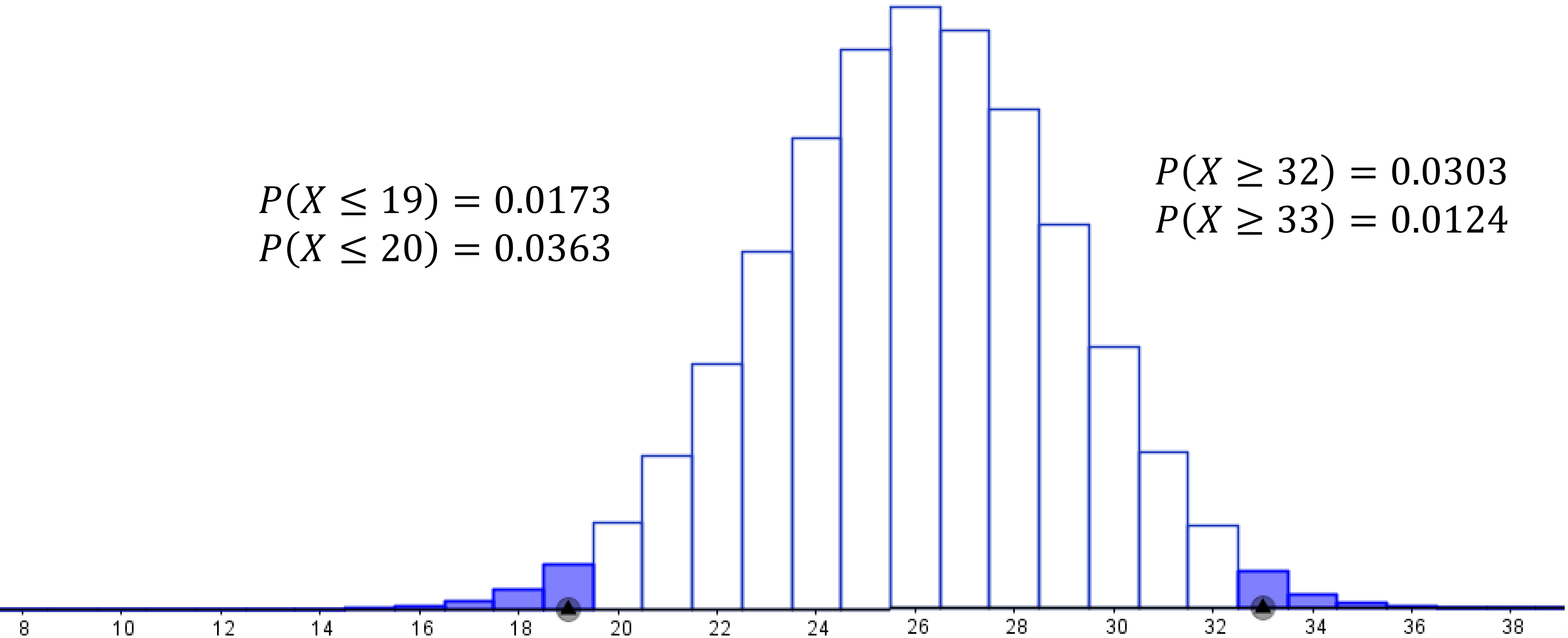
$$P(X \leq 19) = 0.0173$$
$$P(X \leq 20) = 0.0363$$

Critical Regions

- $X \sim$ B(40, 0.65)
- 2.5% at either end of the scale

$P(X \leq 19) = 0.0173$
$P(X \leq 20) = 0.0363$

$P(X \geq 32) = 0.0303$
$P(X \geq 33) = 0.0124$



Critical Regions

- Calculating the critical region

So we reject H_0 if:
$X \leq 19$ or $X \geq 33$

This is the critical region.

$P(X \leq 19) = 0.0173$
$P(X \leq 20) = 0.0363$

$P(X \geq 32) = 0.0303$
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Critical Regions

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$P(X \geq 33) = 0.0124$

- As 19 seeds germinated, we reject H_0 and say that there is evidence at the 5% level for a change in the proportion of seeds germinating

Tea!

A level Mathematics:

The Normal distribution

Aims

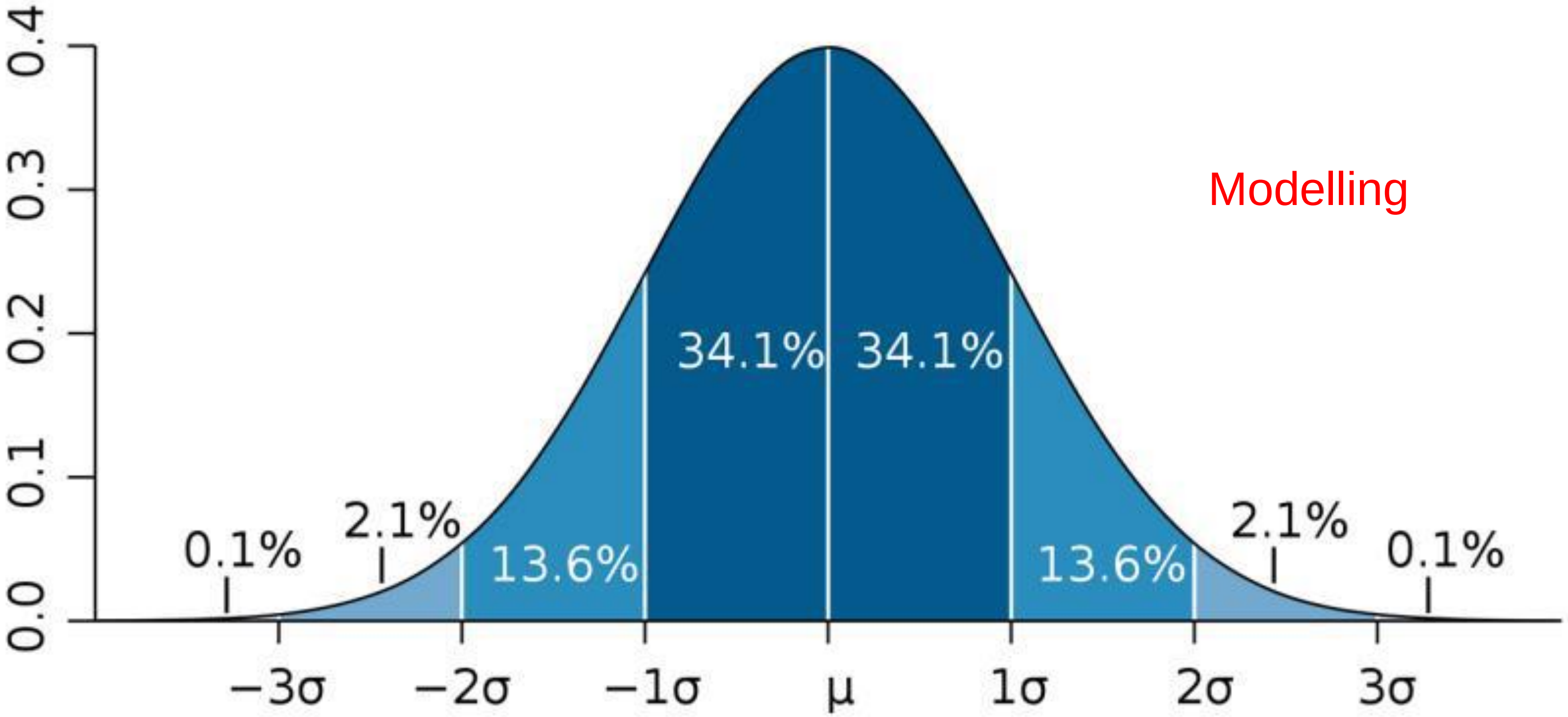
- Use the Normal distribution as a model
- Solve problems that relate to the Normal distribution

Practical Approaches

Ideally you want a sample of least 100 data items:

- Primary Data
  - Leaf Width and Length
  - Heights of people (determine gender/age etc)
  - Weights of a particular denomination of coins
- Secondary Data
  - Take a random sample from a large data set
    - use census at school data – look at heights, weights

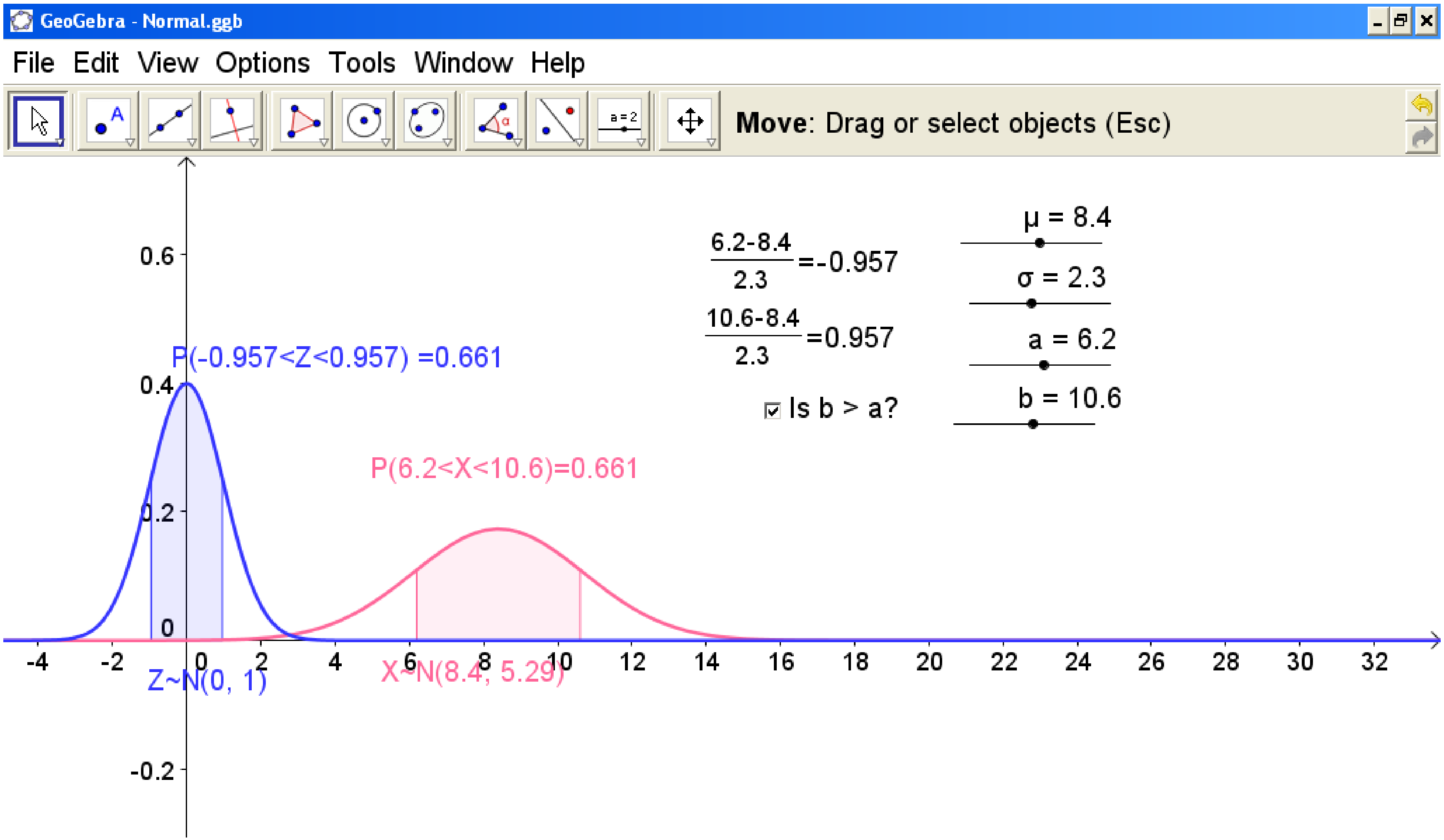
68% of the data lies within one standard deviation of the mean
95% of the data lies within two standard deviations of the mean
99.7% of the data lie within three standard deviations of the mean



Modelling

In theory the curve extends infinitely in both directions, but the chance of finding data items outside the 3 standard deviation mark is very small.

Using *GeoGebra*



Working out Probabilities

What is the probability that a student scores more than 70% in Maths?

What is the probability of a student scoring less than 50% in Maths?

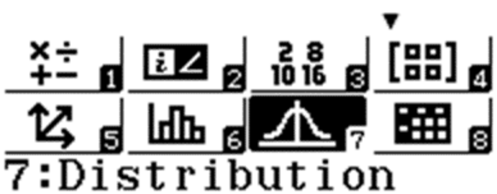
Use of calculator from 2017

Probabilities of types P($X \le x$), P($X < x$), P($X > x$) and P($X \ge x$) can be found directly from the calculator

Example1 (Casio CLASSWIZ – Texas also offering one)

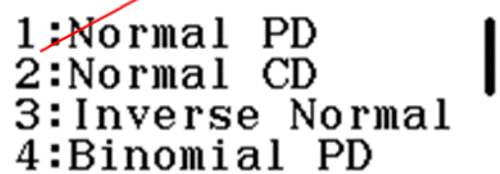
$X \sim$ N(135, 225) Find P($X \le 127$) or P($X < 127$)

Menu

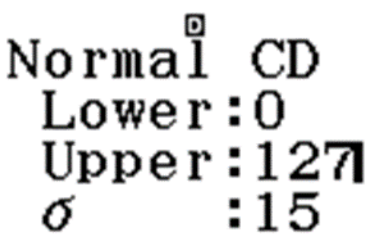


Press 7

Press 2



Select a suitable Lower value – enter info



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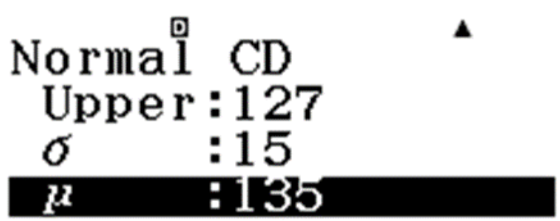
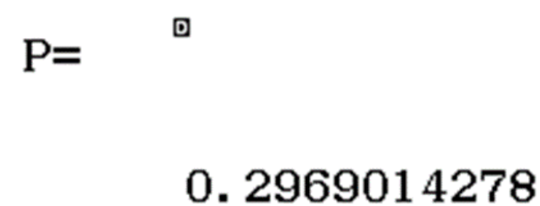




Cost ?

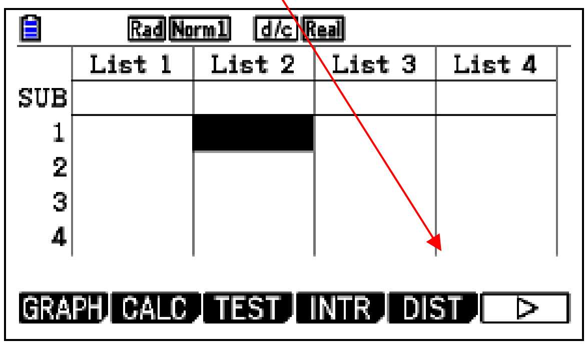
- http://www.studentcalculators.co.uk/acatalog/CLASS_KITS.html

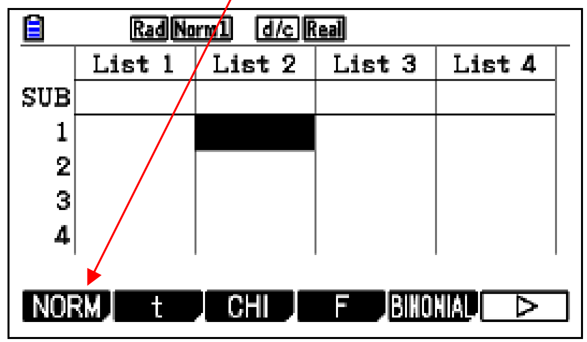
Using CG20 full graphics

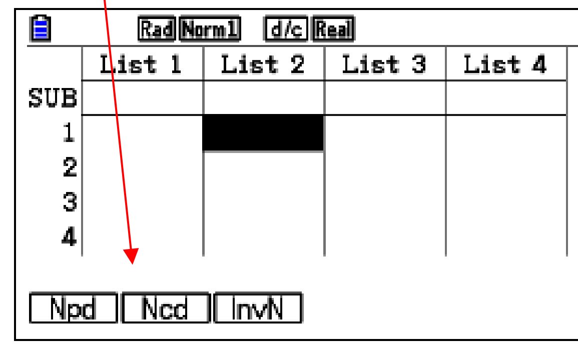
F5 DIST

F1 Norm

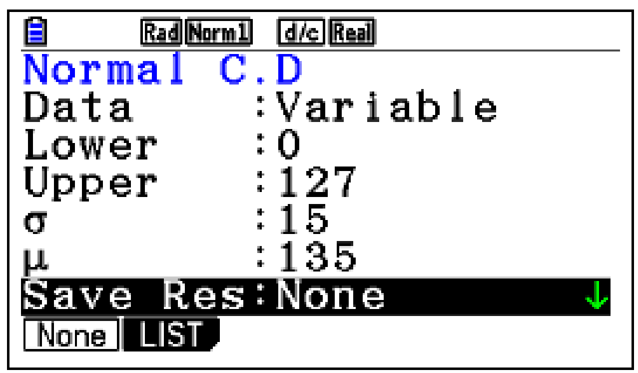
F2 Ncd

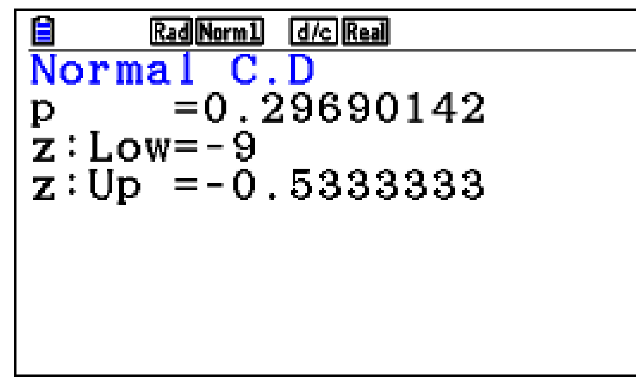




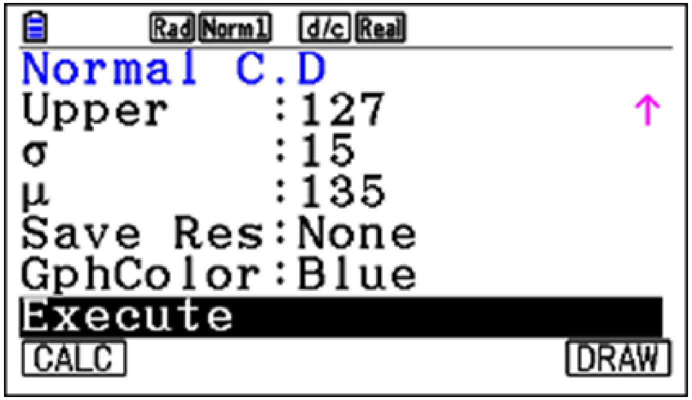


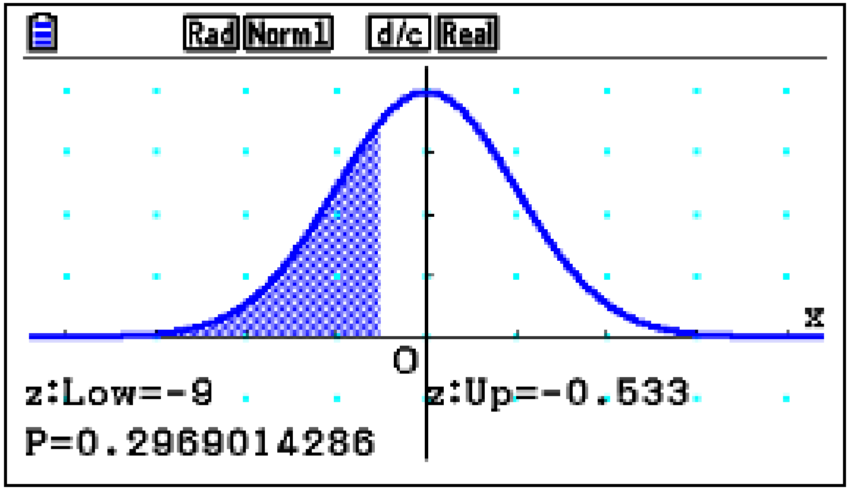
Select a suitable Lower value – enter info





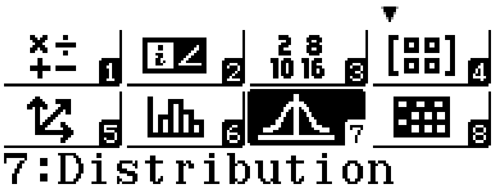
Drawing


Rad Norm1 d/c Real
Normal C.D
Upper :127
σ :15
μ :135
Save Res:None
GphColor:Blue
Execute
CALC
DRAW


Rad Norm1 d/c Real
x
O
z:Low=-9
z:Up=-0.533
P=0.2969014286

Example 2

X ~ Normal mean 135 st deviation 15 P(X ≥ 118) or P(X >118)



Press 7

1:Normal PD
2:Normal CD
3:Inverse Normal
4:Binomial PD

Press 2

Select a suitable Upper value – enter info

Normal CD
Lower:118
Upper:200
σ :15

Normal CD
Upper:200
σ :15
μ :135

Enter data then =

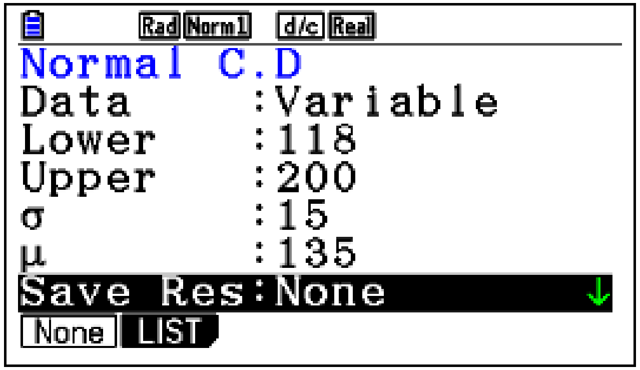
P=

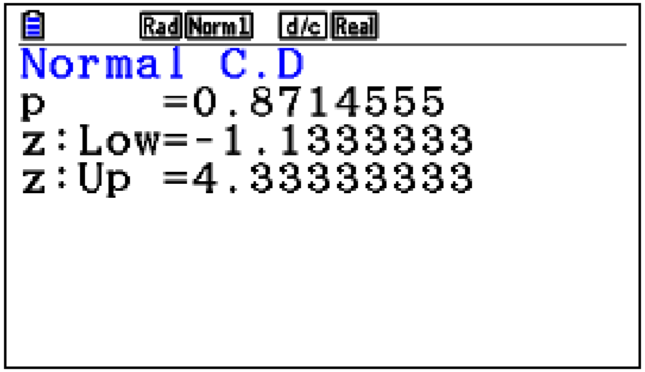
0.8714555072

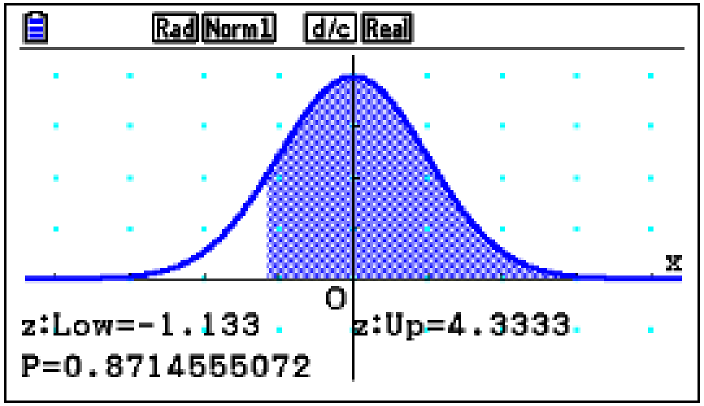
Using CX20 full graphics

X ~ Normal mean 135 st deviation 15 P(X ≥ 118) or P(X >118)

Select a suitable Upper value – enter info







Example 3

X ~ Normal mean 135 st deviation 15 P(119 < X < 128)

Normal CD
Lower:119
Upper:128
σ :15

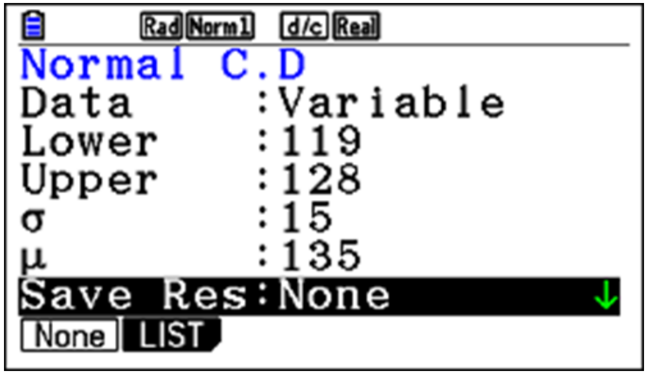
Normal CD
Upper:128
σ :15
μ :135

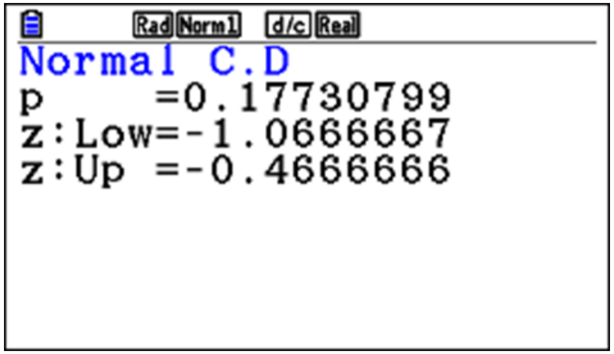
P=

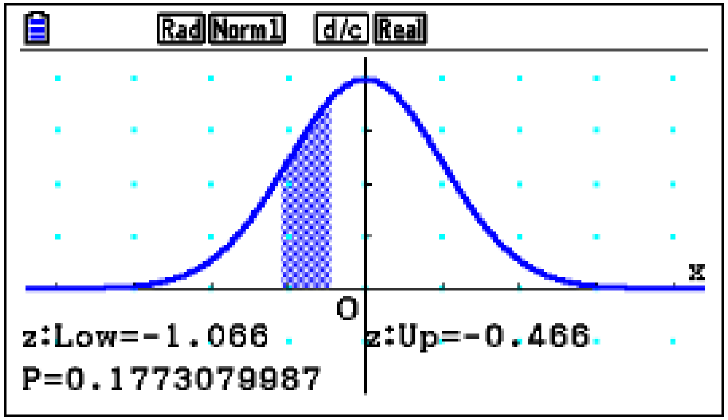
0.1773079985

Using CX20 full graphics

$X \sim$ Normal mean 135 st deviation 15 P(119 < X < 128)







Working out Probabilities

What is the probability that a student scores more than 70% in Maths?

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Maths Scores: M $\sim N(65, 8^2)$

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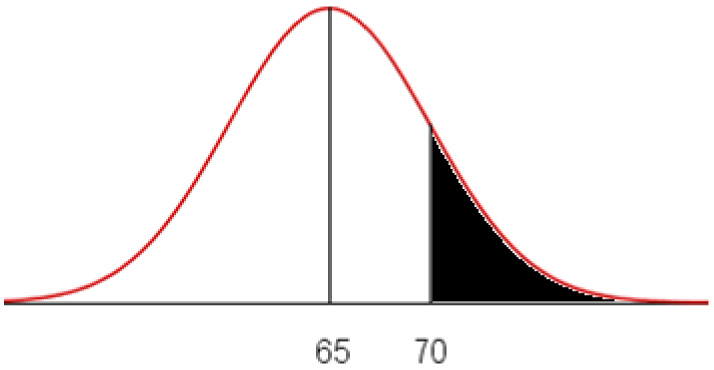
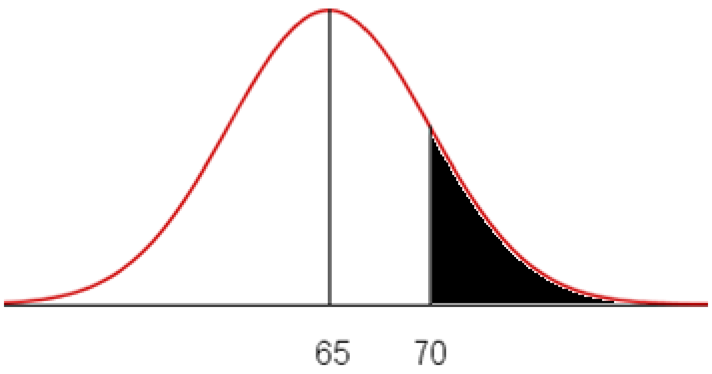


Working out Probabilities

What is the probability that a student scores more than 70% in Maths?

Maths Scores: $M \sim N(65, 8^2)$

$$Z = \frac{70 - 65}{8} = \frac{5}{8} = 0.625$$



Working out Probabilities

What is the probability that a student scores more than 70% in Maths? P(X >70)

Maths Scores: $M \sim N(65, 8^2)$

$$Z = \frac{70-65}{8} = \frac{5}{8} = 0.625$$

What is the probability that a student scores more than 70% in Maths? P(X >70)

Maths Scores: $M \sim N(65, 8^2)$

$$Z = \frac{70-65}{8} = \frac{5}{8} = 0.625$$

So we need P(Z >0.625) where $Z \sim N(0, 1^2)$

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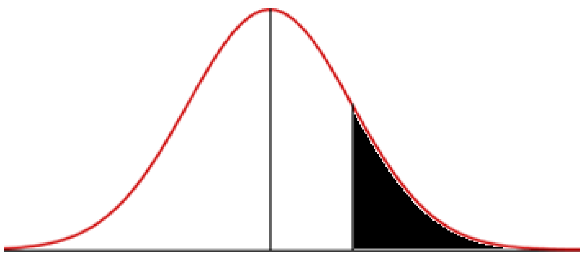
Working out Probabilities

P(X >70) Maths Scores: $M \sim N(65, 8^2)$

$$Z = \frac{70-65}{8} = \frac{5}{8} = 0.625$$



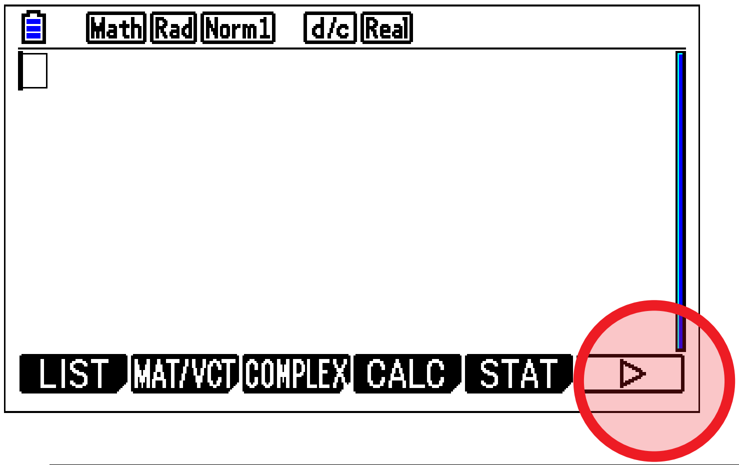
So we need P(Z >0.625) where $Z \sim N(0, 1^2)$

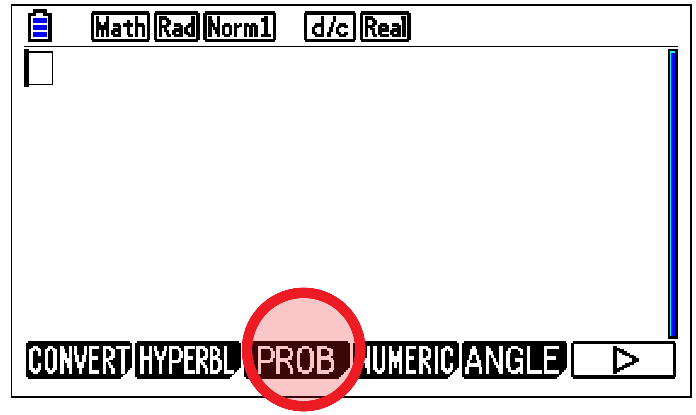
We can use a calculator directly to calculate this once we have the Z value.

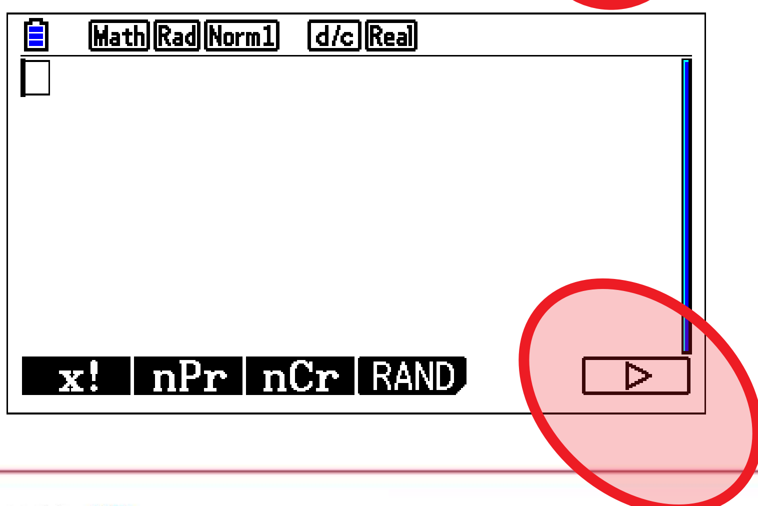
$Z \sim N(0,1^2)$ is programmed into the calculator

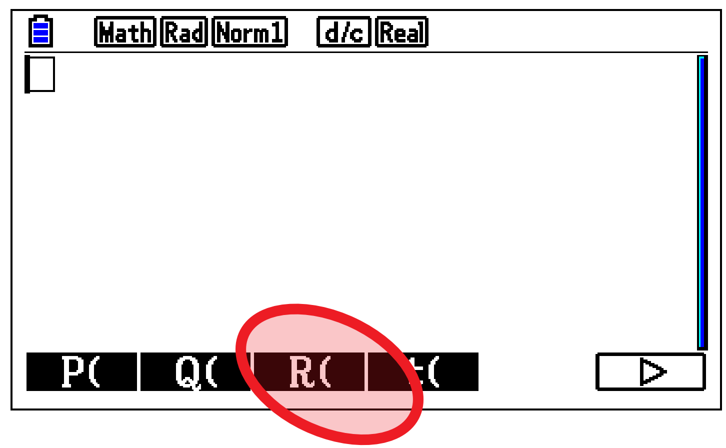
Have the calculator in ‘Run’ mode.

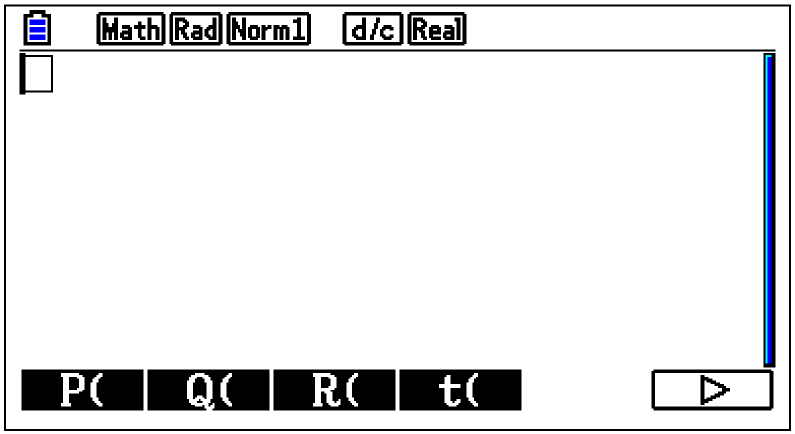
Press the OPT key

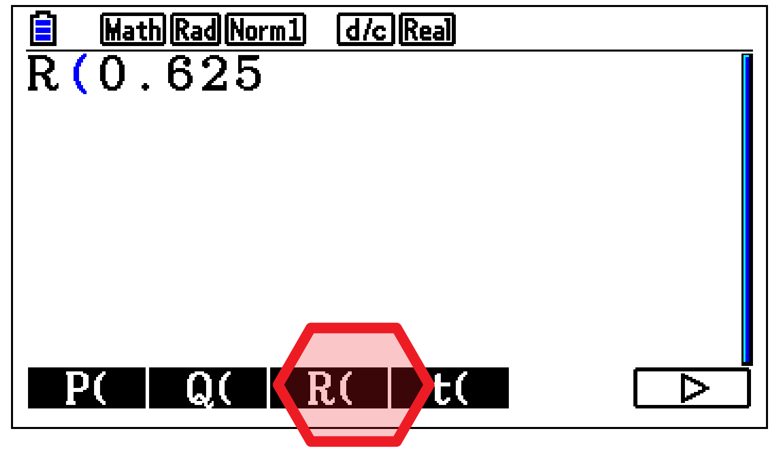


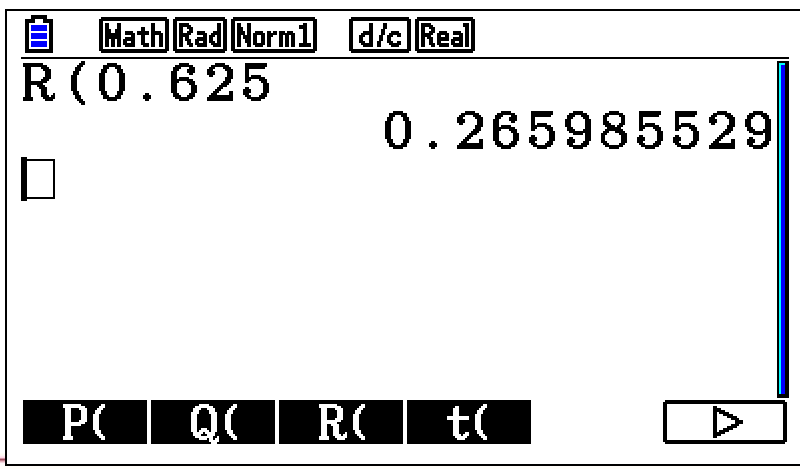










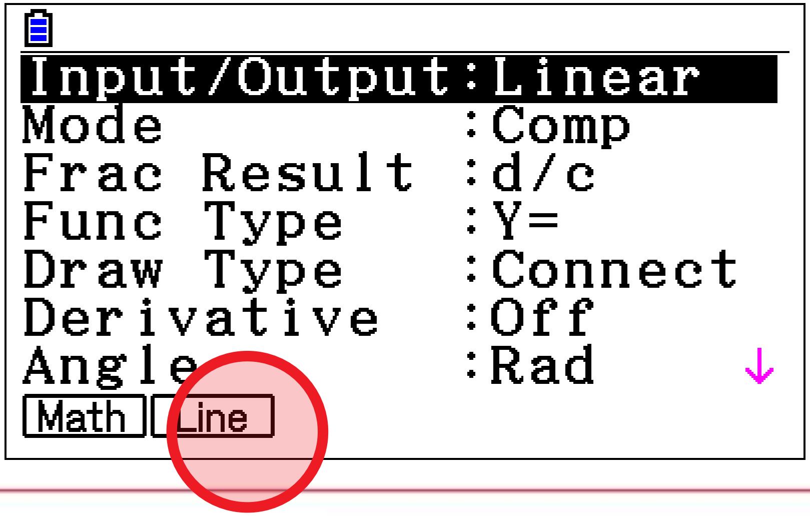


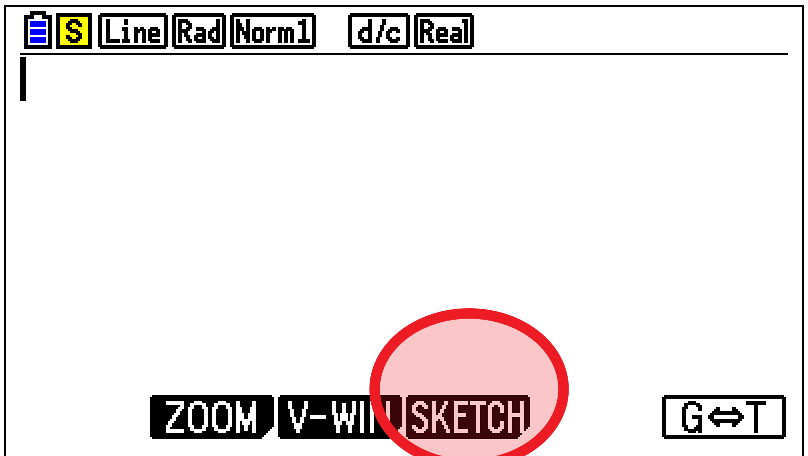
P is P(Z<z)

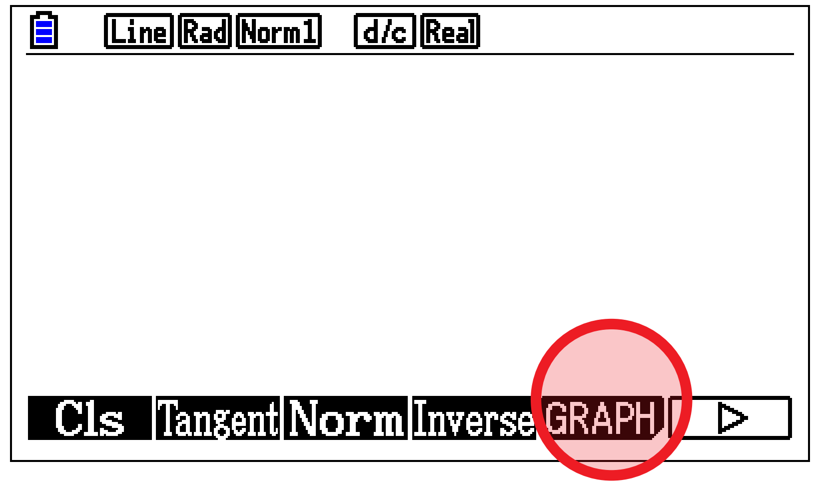
R is P(Z>z)

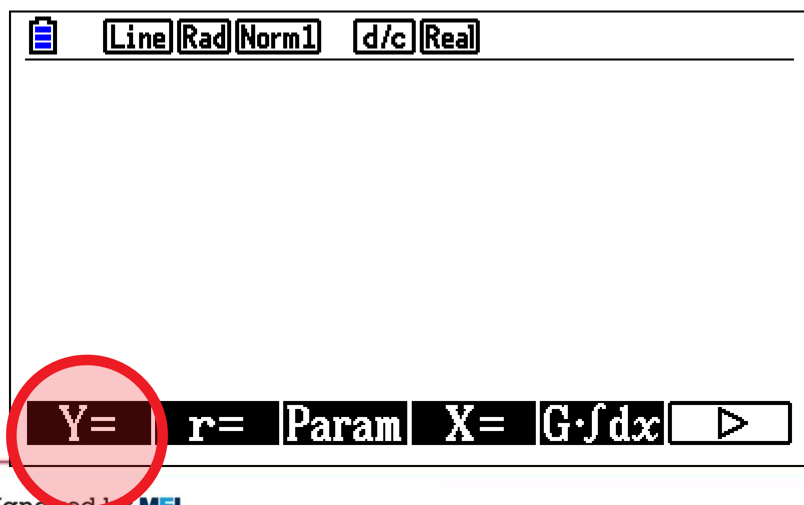
Q is P(Z is between 0 and z)

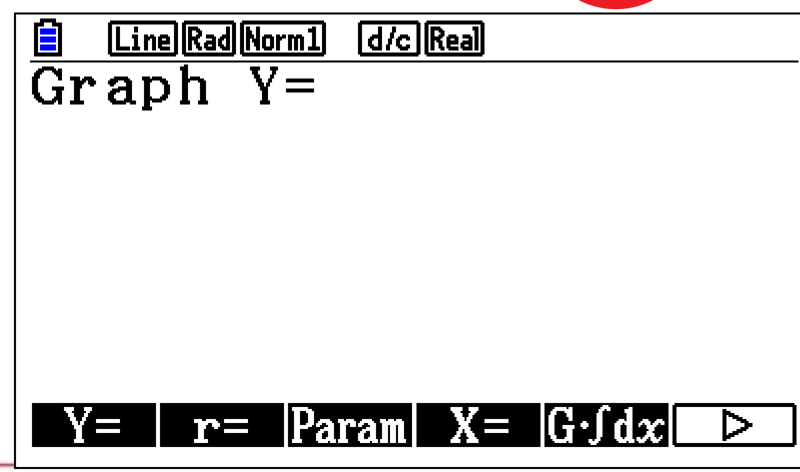
- We can also graph this directly
- Menu Run
- Shift Menu F2 EXIT
- Input/output change to Linear, then EXIT



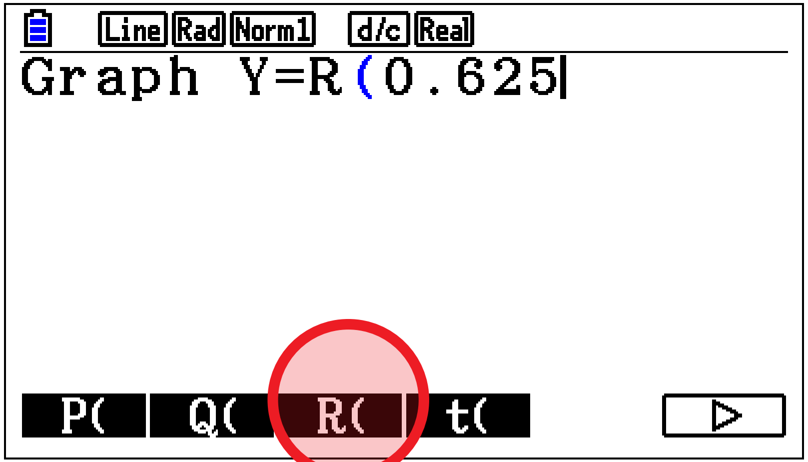

S Line Rad Norm1 d/c Real
ZOOM V-WIN SKETCH G⇔T


Line Rad Norm1 d/c Real
Cls Tangent Norm Inverse GRAPH ▷

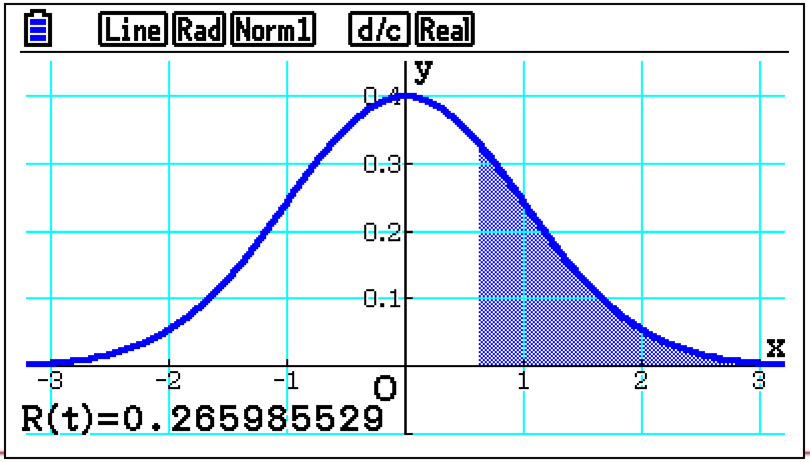

Line Rad Norm1 d/c Real
Y= r= Param X= G·∫dx ▷


Line Rad Norm1 d/c Real
Graph Y=
Y= r= Param X= G·∫dx ▷

To get to the following screen:



Then press EXE



Question

What is the probability of a student scoring less than 50% in Maths? $P(X < 50)$

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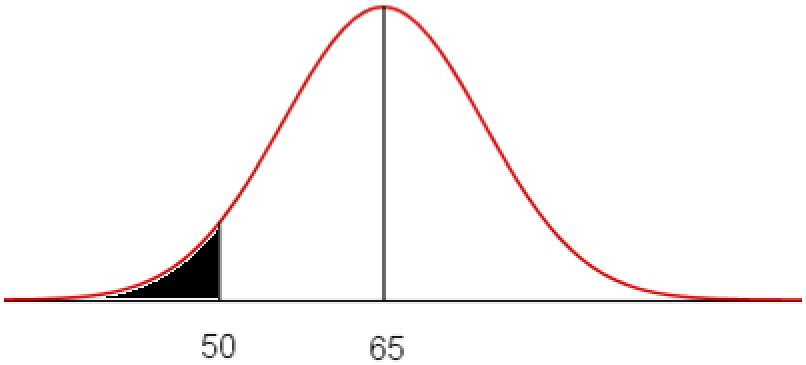
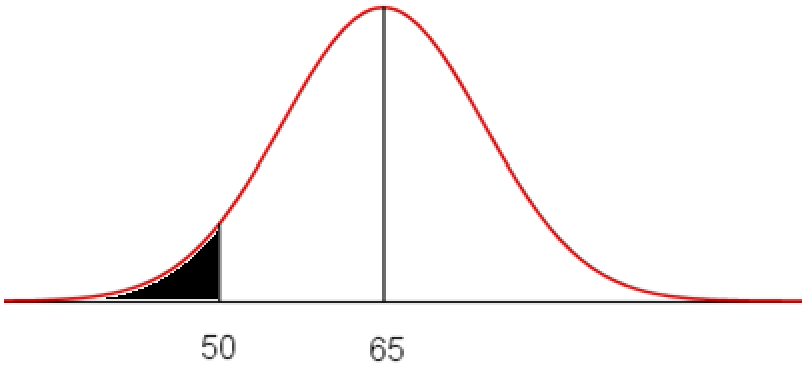


Question

What is the probability of a student scoring less than 50% in Maths?

$$Z = \frac{50-65}{8} = \frac{-15}{8} = -1.875$$



Question

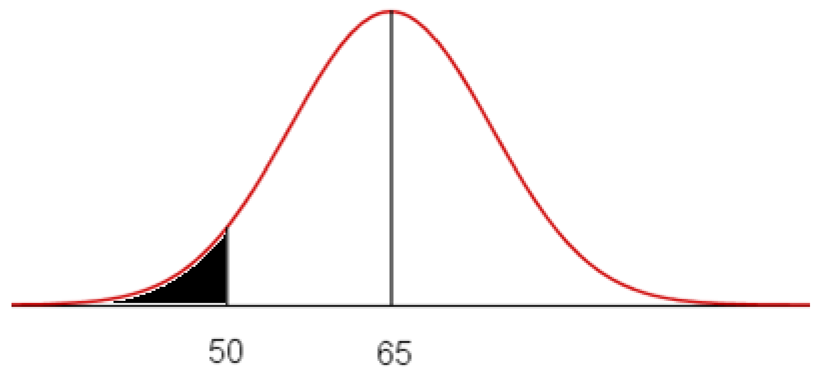
What is the probability of a student scoring less than 50% in Maths?

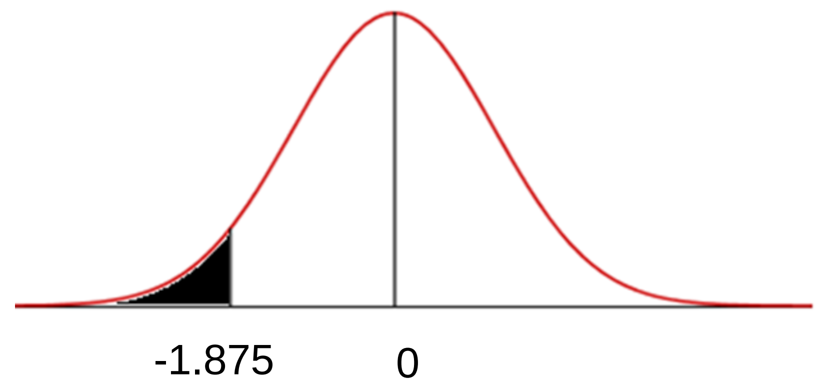
$$Z = \frac{50-65}{8} = \frac{-15}{8} = -1.875$$

Using a calculator:

P(Z < -1.875) = 0.0303

or P(X <50) = 0.0303 directly





Question

What is the probability of a student scoring less than 50% in Maths?

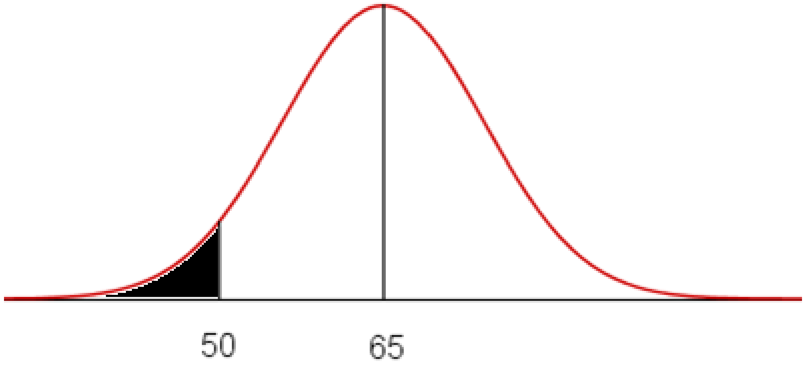
$$Z = \frac{50-65}{8} = \frac{-15}{8} = -1.875$$

Using tables - have to use the

symmetry of the curve and calculate

$\Phi(1.875) = P(Z < 1.875) = 0.9697$

Hence $P(Z < -1.875) = 1 - 0.9697$



Awkward so use calculator

Question

What is the probability of a student scoring between 45% and 55% in Maths?

$Z_1 = \frac{55-65}{8} = \frac{-10}{8} = -1.25$ $\quad Z_2 = \frac{45-65}{8} = -2.5$

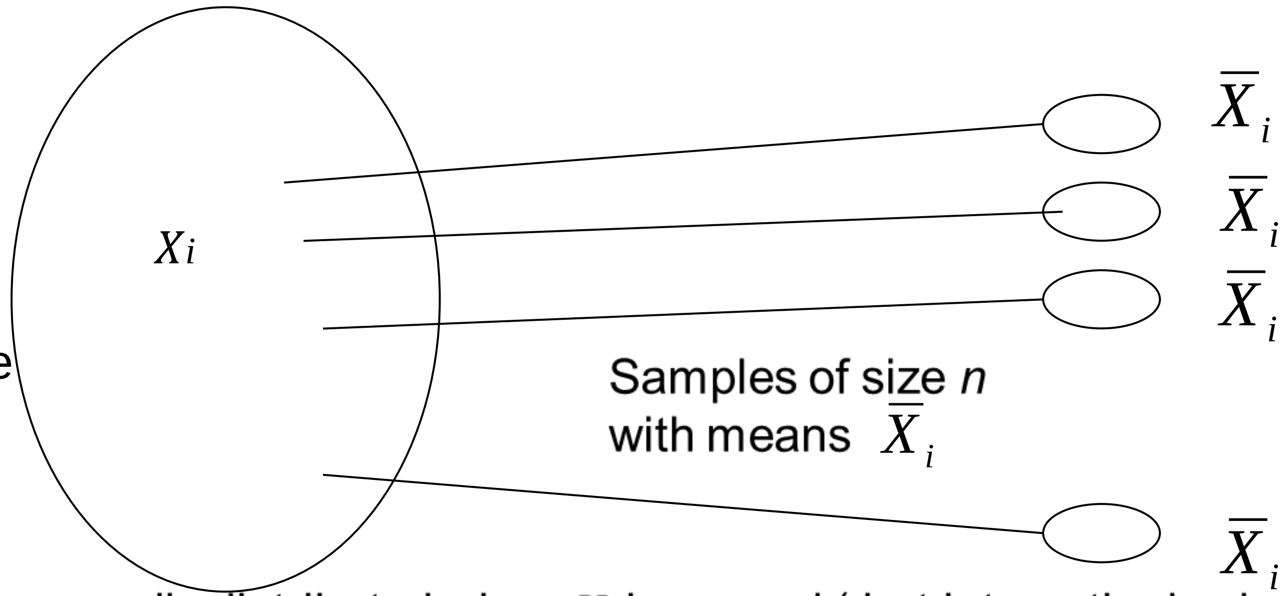
Use the calculator to calculate directly

P(-2.5 < Z < -1.25) = P(Z < *-1.25*) – P(Z < -2.5) = 0.099403

Hypothesis testing with normal distribution

Illustration of distribution of MEANS found from S**AMPLES** taken from a big **POPULATION**

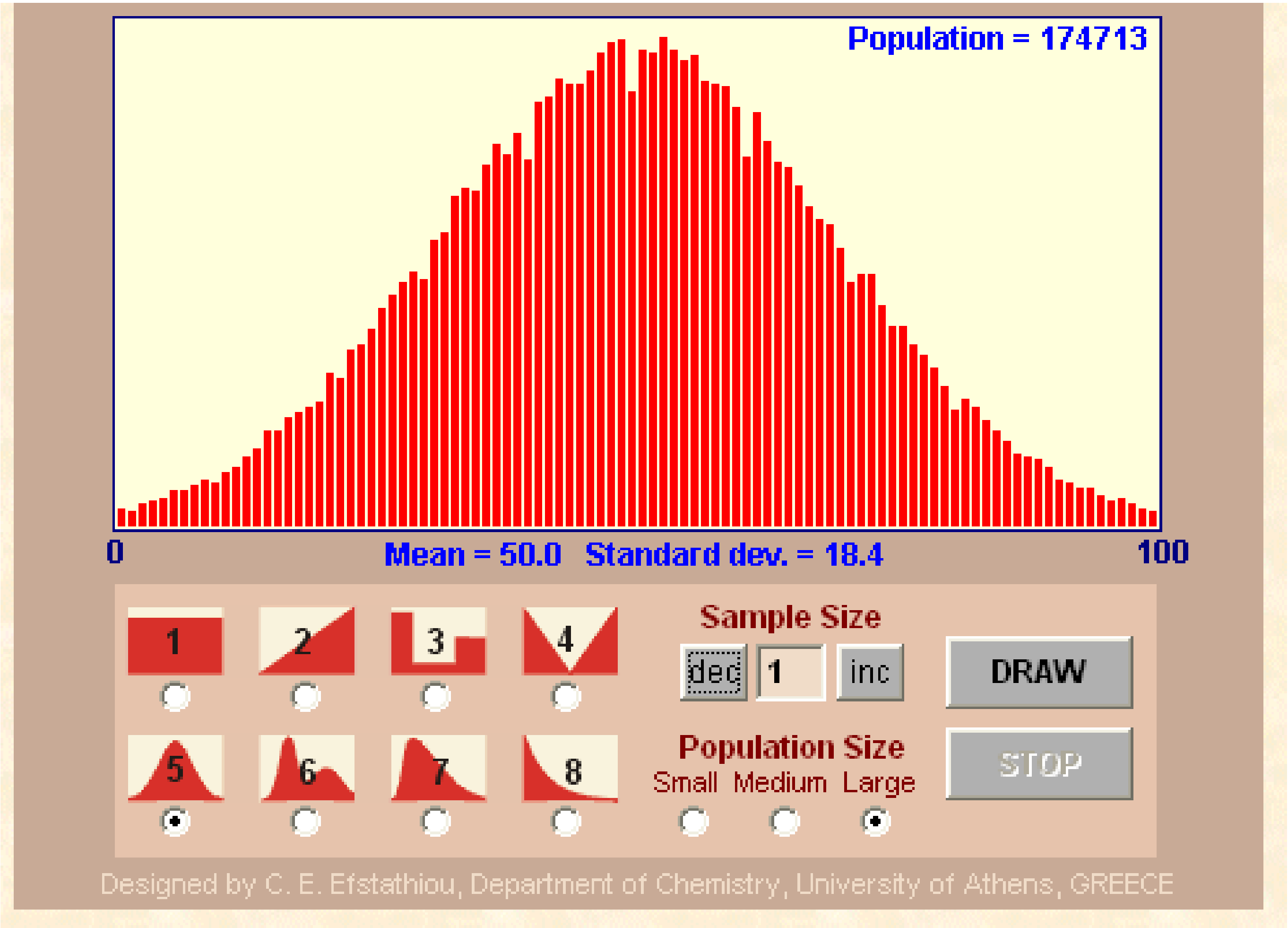
Population with mean μ and variance σ^2



The $\overline{X}_i$ are normally distributed when X is normal (but interestingly also always normally distributed, regardless of the distribution of X provided that n, the sample size, is ≥ 30.)

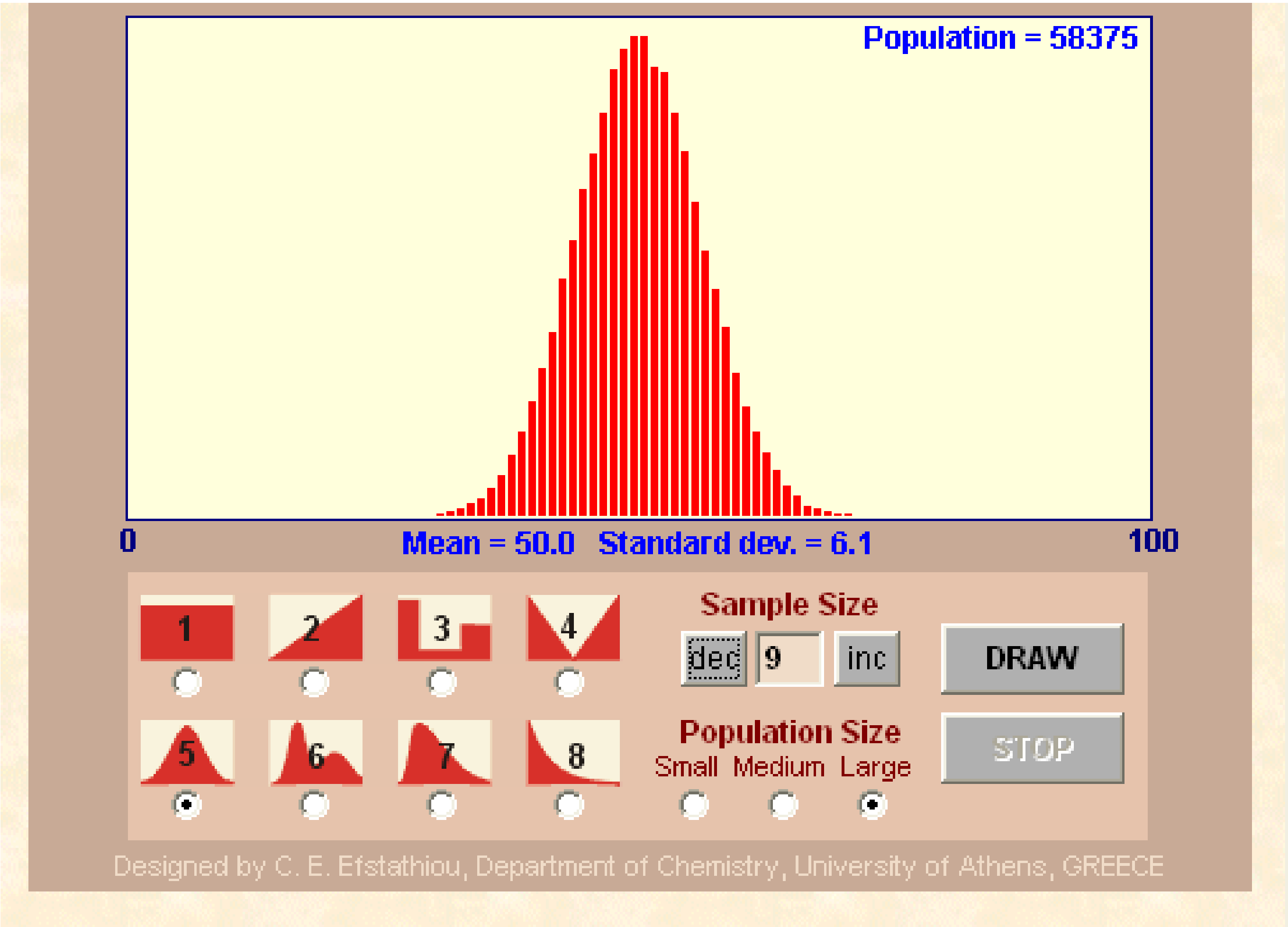
$$\overline{X}_i \sim \mathrm{N}(\mu, \sigma^2/n).$$

Population



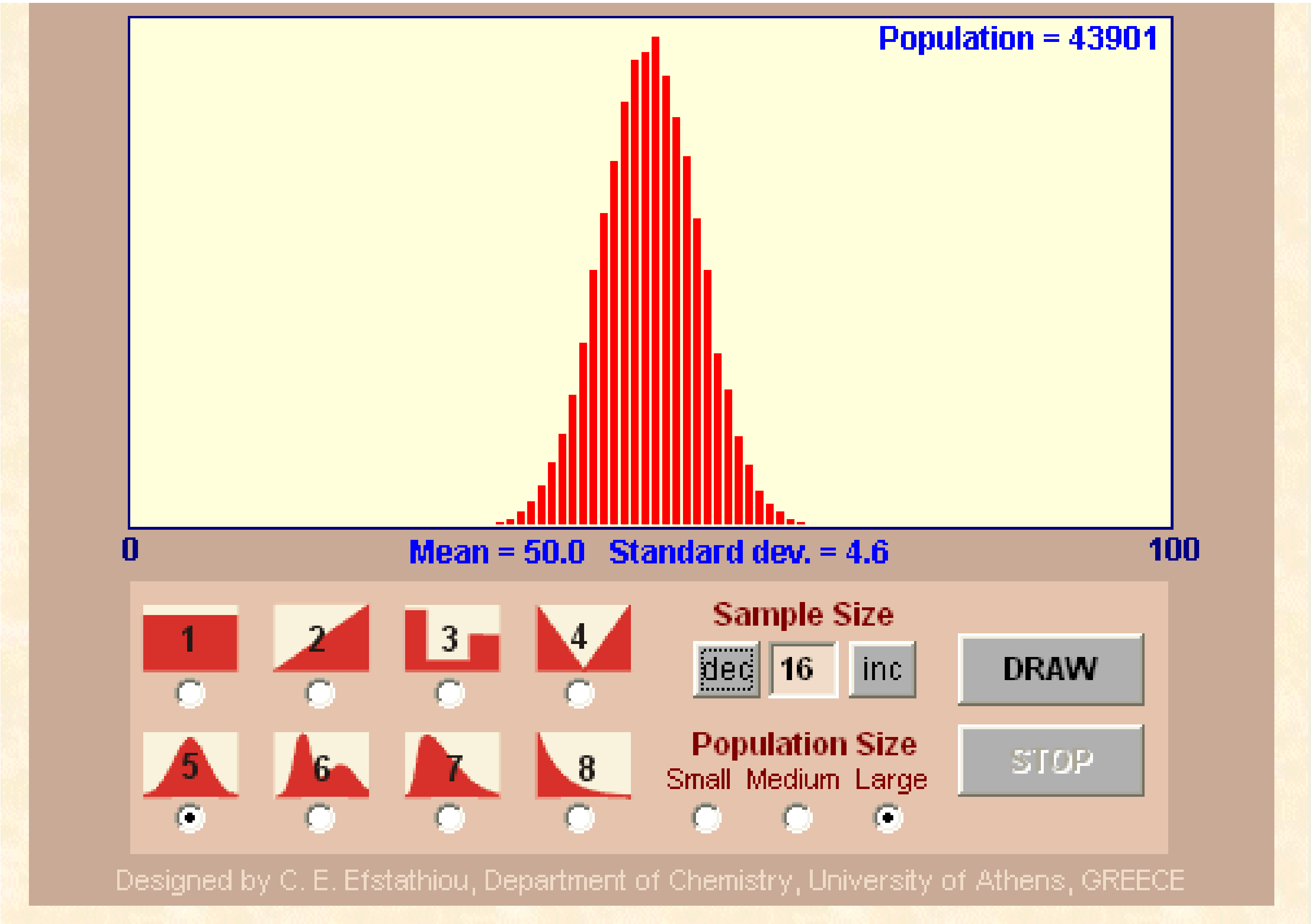
Parent population: Normal Distribution

Distribution of sample means



Distribution of sample means, size 9

Distribution of sample means



Distribution of sample means, size 16

Why is the Normal Distribution so important?

- Common occurrence in nature, science and manufacturing processes
- Often can assume that the distribution is approximately normal e.g. IQ scores
- Central Limit Theorem says that if we take samples of sufficiently large size the means of these samples are normally distributed, regardless of the original distribution.
- Because of this the normal distribution underpins all statistical inferences about means and proportions.

Hypothesis Testing – Normal Distribution

A machine is designed to make paper clips with mean mass 4.00 g and standard deviation 0.08 g.

Masses of paperclips are normally distributed.

A quality control officer weighs a *random sample* of 25 paper clips and finds that the mean mass: $\bar{X} = 4.04$ g.

- Let X represent the mass of a paperclip.
- Let μ represent the population mean mass of paper clips and σ represent the standard deviation of the masses of paper clips.
- Then X is a normal variable with distribution: $X \sim N(\mu, \sigma^2)$
- Assuming that σ is unchanged at 0.08 g, then $X \sim N(\mu, 0.08^2)$

What about μ ? Is it still 4.00 g or is there sufficient evidence to suggest that it has increased?

Hypothesis Testing – Normal Distribution

Firstly formalise the question by setting up two hypotheses :

- **Null** hypothesis H_0: $\mu = 4.00$

  (no change in population mean μ)

- **Alternative** hypothesis H_1: $\mu > 4.00$

  (population mean μ has increased)

- The sample mean, $\bar{X}$, that the quality control officer calculates, becomes the **test statistic**.
- Assuming that H_0 is true, calculate $\mathrm{P}(\bar{X} > 4.04)$ and compare it with the chosen **significance level** , say 1% or 0.01.
- If $\mathrm{P}(\bar{X} > 4.04) < 0.01$ then reject H_0 in favour of H_1.
- If $\mathrm{P}(\bar{X} > 4.04) > 0.01$ then *do not* reject H_0 in favour of H_1.

In order to do the calculations we must use the distribution of $\bar{X}$

From before we know that if $X \sim N(\mu, \sigma^2)$

Then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ where n is the sample size

So we use: $\bar{X} \sim N(4.04, \frac{0.08^2}{25})$

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- If $P(\bar{X} > 4.04) > 0.01$ then *do not* reject H_0 in favour of H_1.

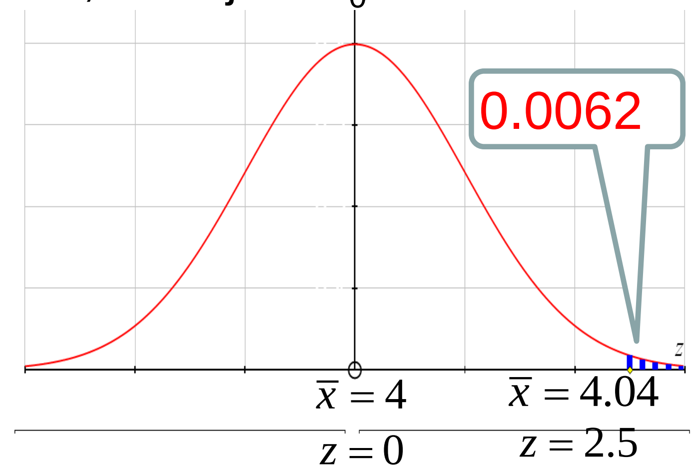
Assuming that H_0 is true, X ~ N(4.00, 0.08²):

$$P(\bar{X} > 4.04) = P\left(Z > \frac{4.04 - 4.00}{0.08/\sqrt{25}}\right)$$

Can use calculator

$$= P(Z > 2.5) = 1 - P(Z \le 2.5) = 1 - 0.9938 = \mathbf{0.0062}$$

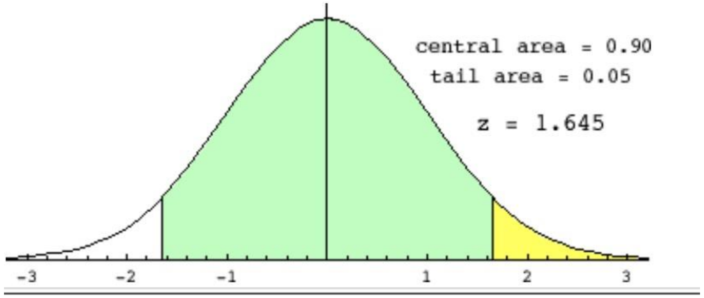
Since $P(\bar{X} > 4.04)$ = **0.0062 < 0.01**, we reject H_0 in favour of H_1 at the 1% significance level.

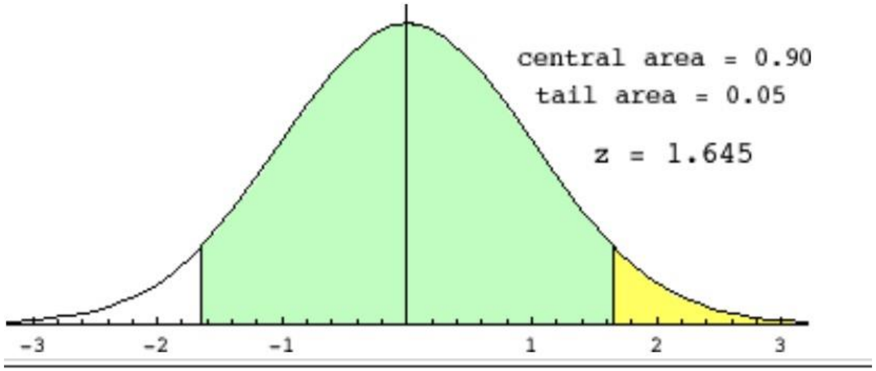


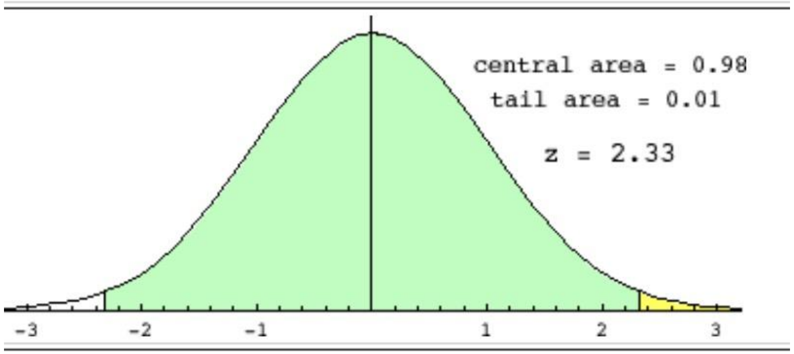
The result is *significant* – there is sufficient evidence to suggest that the population mean mass of paper clips has increased.

Hypothesis Testing – Normal Distribution *alternative approach*

We use information about probabilities associated with certain values of Z. These are called **CRITICAL VALUES**





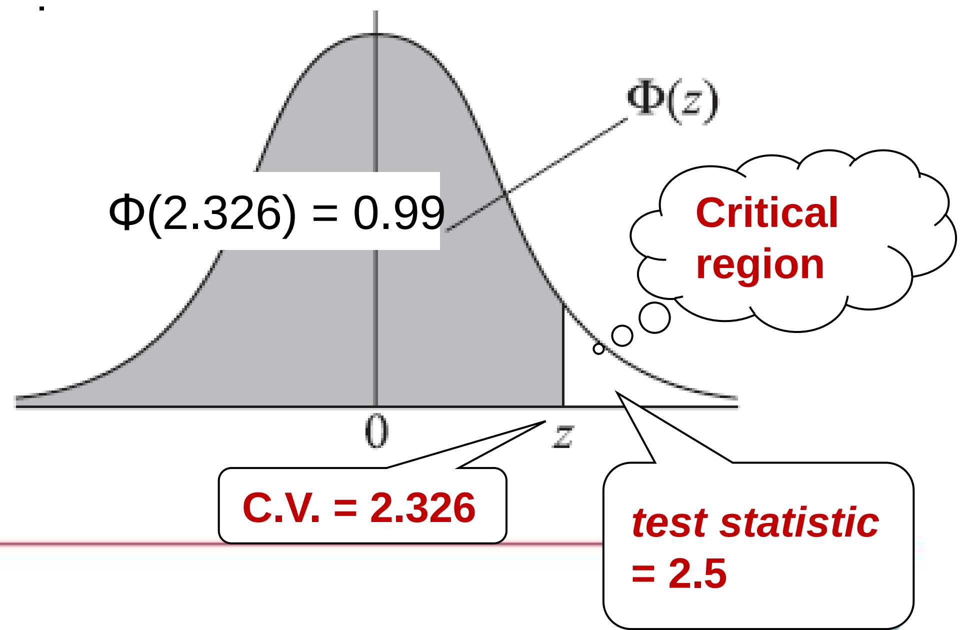


Hypothesis Testing – Normal Distribution *alternative approach*

$$\bar{x} = 4.04 \quad \Rightarrow \quad z = \frac{4.04 - 4.00}{0.08 / \sqrt{25}} = 2.5$$

Find the critical z-value corresponding to a 1% significance level and compare with calculated z-value, the **test statistic.**

Since **2.5 > 2.326** , we reject H_0 in favour of H_1 at the 1% significance le



The result is *significant* – there is sufficient evidence to suggest that the there is sufficient evidence to suggest that the population mean mass of paper clips has increased.

.

Summary: *The five steps for the hypothesis test*

Let X represent the mass of a paperclip, masses such that $X \sim \mathrm{N}(\mu, 0.08^2)$:

(1) Establish null and alternative hypotheses:

H_0: $\mu = 4.00$ H_1: $\mu > 4.00$ (one-tail test)

(2) Significance level, *significance level* = 1% = 0.01

(3) Find measures: sample mean = 4.04

sample size, $n = 25$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{4.04 - 4.00}{0.08/\sqrt{25}} = 2.5$$

(4) Conduct test:

Critical value = $\Phi^{-1}(99\%) = 2.326$, and 2.5 **>** 2.326

or p value is 0.0062 < 0.01

(5) Interpret result in terms of the original context:

Since 2.5 **>** 2.326, or since 0.0062 < 0.01, reject H_0. There is sufficient evidence to suggest that the population mean of paperclips has increased.

Hypothesis Testing for μ: $X \sim \mathrm{N}(\mu, \sigma^2)$ – two tail test

(1) Establish null and alternative hypotheses:

H$_0$: $\mu = \mu_0$ H$_1$: $\mu \neq \mu_0$

(2) Decide on the significance level: *sig* α%

(3) Collect data and find measures from random sample size n, taken from population; Calculate:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

(4) Conduct test:
Compare $|z|$ with critical value [$\Phi^{-1}([100 - ½\alpha]\%)$]

or p value with α% (on graphics calc)

(5) Interpret result in terms of the original claim:

If $|z|$ > critical value [$\Phi^{-1}([100 - ½\alpha]\%)$] or $p < ½\alpha\%$ then reject H$_0$ and write conclusion in context of problem.

Some practical suggestions for Hypothesis tests in the classroom

- **Number of Skittles in a bag – does the average number match the manufacturer's claims?**

Some practical suggestions for Hypothesis tests in the classroom

- **Reaction Times – does the time taken to catch a reaction ruler with the non-dominant hand differ from that of the dominant hand?**
- https://backyardbrains.com/experiments/reactiontime
- http://www.humanbenchmark.com/tests/reactiontime/statistics
- https://backyardbrains.com/experiments/files/Sensory%20Reflex%20Handout%20by%20Virginia%20Johnson.pdf

Some practical suggestions for Hypothesis tests in the classroom

- **Newspaper articles – does an article come from a newspaper, by comparing sentence length?**

Problems to be aware of

- **We don’t often know the population variance, so will have to estimate it from the sample**

s^2

- **We may not know the population mean, so will have to calculate it first (or at least get some estimate of it)**

- **This means that our methods are not strictly accurate.**

A level Mathematics: sampling

Aims:

- Understand why samples would be carried out
- Know some sampling methods
- Understand the limitations of sampling

Sampling content

- **Understand and use sampling techniques, including simple random sampling and opportunity sampling**
- **Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population**

Why sample?

- Before computers did calculations, it was more sensible to take a sample to find a summary statistic
- Now spreadsheet software can calculate most summary statistics… why sample?
- It is important to know if the summary statistic is representative of the whole data set
  - Which average wage would be the best to calculate for company where twenty employees are on minimum wage and the one director earns £500,000?

Is it representative?

- In the last example the most representative average was the median…however, it is not always that obvious, nor are the differences always that clear
- If you take repeated samples and calculate the same statistic for each sample, if the data set is uniform then the statistic will remain reasonably constant (**not** exactly the same)
- If in repeated samples the statistic changes dramatically, then the data set has some non-uniform information and the summary statistics for the whole date set should only be used with care.

A level Mathematics:

large data sets

Aims:

- Understand why Large Data Sets (LDS) have been introduced at A level
- Explore some activities using LDS
- Consider ways to use LDS in the classroom as a teaching tool

Large Data Sets

- What are they?
- Why are they important?
- What issues do they create?

Large Data Sets

- What are they?
- Why are they important?
- What issues do thy create?
- How do we deal with these?

Thinking about Data

- Using the data provided, how could students analyse it?

- What questions could students be asked to answer?

What techniques at AS could we use?

- Sampling
- Histograms
- Scatter graphs and correlation (not causation)
- Measures of central tendency and spread (standard deviation)
- Select and critique different presentation techniques
- Probability: exclusive and independent events

Analysing data

- Question: Are fewer babies being born?
- Rationale: Lots of people I know only have one child.

- Use the diagrams and calculations provided to answer the question: Are fewer babies being born?

Types of Questions

- Short with brief interpretation
- Deep interpretation of the data, using given graphs and summaries
- Selection from given graphs and summary data
- Modelling with trend lines for bivariate data
- Modelling with distributions and hypothesis testing
- Describing a situation where data needed to be collected and how it might be done

Data Sets – draft specifications

Body	Format	Description of data	Comment	Lifetime
AQA	PDF gives links to 40+ datasheets on gov.uk site	Family Food datasets	Most have no categorical fields	Until further notice
Edexcel	2 sets of 5 spreadsheets	Met Office weather for 5 different stations over 2 different time periods	No categorical fields and nothing to explain the data	Until further notice
MEI	Single sheet spreadsheet	2012 Olympics Medals and demographic data by country	2 categorical fields	New data set for each cohort
OCR	Spreadsheet with 4 sheets	Methods of Travel by Local Authority	2 categorical fields	Until further notice

AQA Data Set

Table 2.4 UK eating out purchased quantities of food and drink

		2010	2011	2012	2013	2014
Number of households in sample		5263	5692	5596	5144	5134
Number of persons in sample		12196	13448	13196	12144	12150
Eating Out Purchases						
Alcoholic drinks						
average across whole population	ml	413	394	355	321	320
average excluding under 14's	ml	494	472	426	386	320
Soft drinks inc. milk drinks	ml	279	269	254	264	267
Other food products [c]		144	118	103	107	117
Beverages	ml	117	117	118	115	118

AQA Data Set

Table 2.4 UK eating out purchased quantities of food and drink

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Other food products (c)		144	118	103	107	117
Beverages	ml	117	117	118	[illegible]5	118

Is this right?

AQA Data Set

I wonder which is correct?

Table 2.4 UK eating out purchased quant

		2010	[illegible]	[illegible]	2013	2014
Number of households in sample		5263	5692	5596	5144	5134
Number of persons in sample		12196	13448	13196	12[illegible]44	12150
Eating Out Purchases						
Alcoholic drinks						
average across whole population	ml	413	394	355	321	320
average excluding under 14's	ml	494	472	426	386	320
Soft drinks inc. milk drinks	ml	279	269	254	264	267
Other food products [(c)]		144	118	103	107	117
Beverages	ml	117	117	118	[illegible]5	118

Is this right?

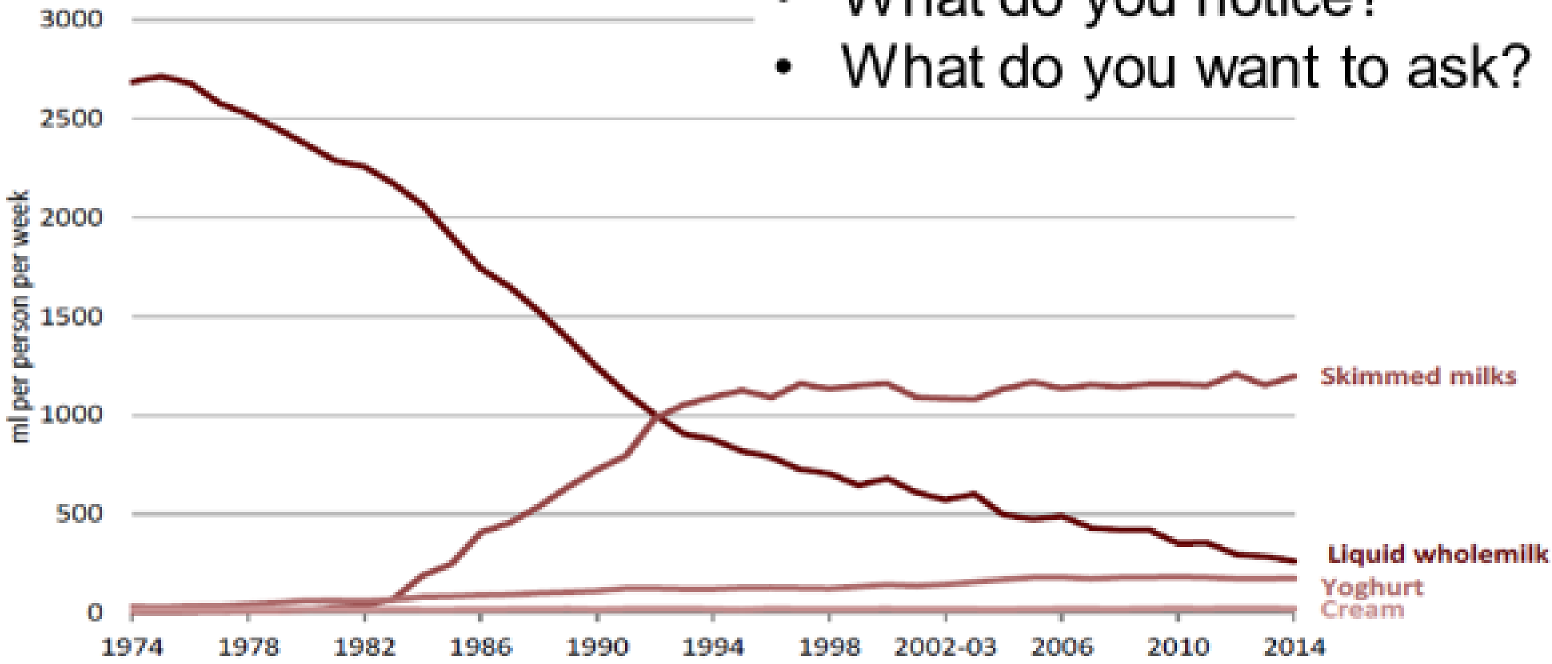
AQA Data Set


ml
Alcohol eating out - average per person per week
600
500
400
300
200
100
0
2009
2010
2011
2012
2013
2014
2015
Whole population
Excluding under 14s

Starting with a diagram

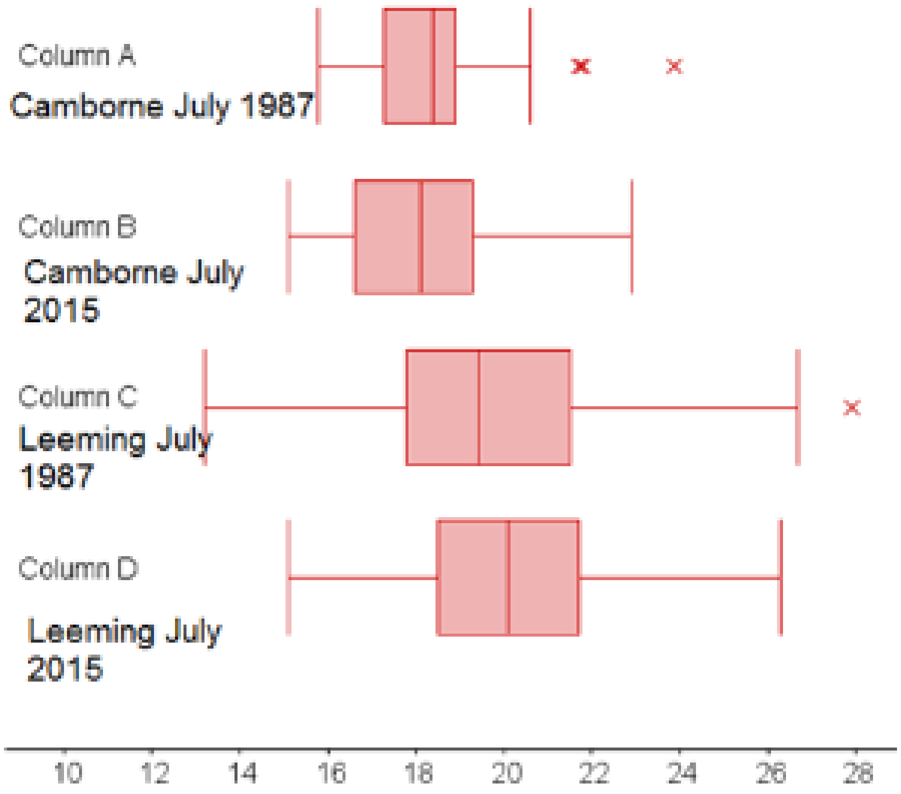
Chart 2.1: UK purchases of milk and milk products, 1974 – 2014

- What do you notice?
- What do you want to ask?



Edexcel Data Set

- Maximum Temperatures in July, drawn with GeoGebra



MEI Dataset: Correlation

More support

- https://integralmaths.org/statistics.php

AS vs GCSE

Students find this transition difficult, many techniques are shared, but critical are:

- Use of appropriate calculations and comparisons

AS vs GCSE

Students find this transition difficult, many techniques are shared, but critical are:

- Use of appropriate calculations and comparisons
- Depth of answer

AS vs GCSE

Students find this transition difficult, many techniques are shared, but critical are:

- Use of appropriate calculations and comparisons
- Depth of answer
    - dealing with analysis in CONTEXT
    - examining the SIGNIFICANCE

A real accredited sample paper!

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \dots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \dots\ + b^n \qquad (n \in \square),$$

where $${}^n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \dots\ + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \ \dots \qquad (|x|<1,\ \ n \in \square)$$

A real accredited sample paper!

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B) \quad \textbf{or} \quad P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

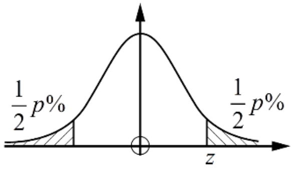
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.575



A real accredited sample paper!

5 In a particular country, 8% of the population has blue eyes. Find the probability that, in a random sample of 20 people, exactly two have blue eyes. **[2]**

5			Binomial(20, 0.08) P(2 blue) = 0.27[11]	**M1** **A1** **[2]**	**3.3** **1.1**	**BC**

6 Each day, for many years, the maximum temperature in degrees Celsius at a particular location is recorded. The maximum temperatures for days in October can be modelled by a Normal distribution; the appropriate Normal curve is shown in Fig. 6.

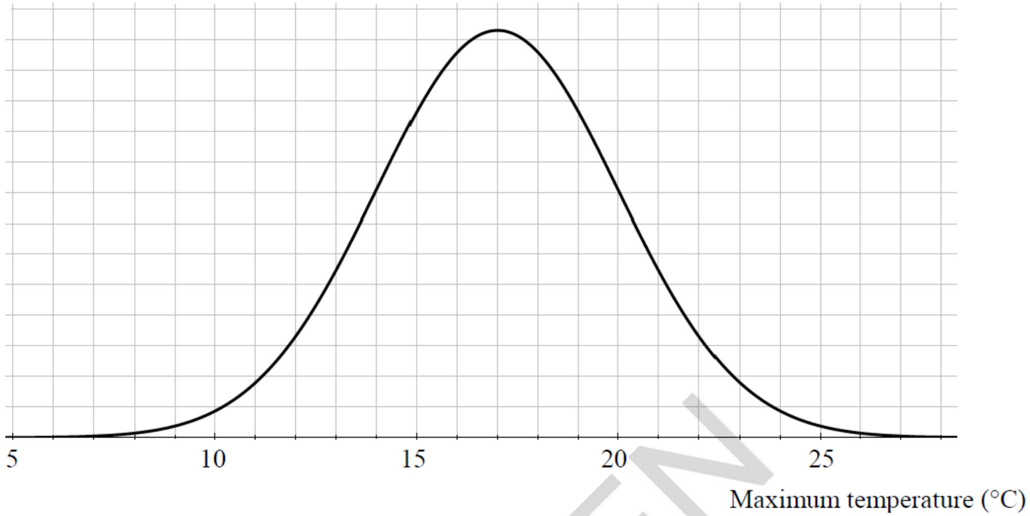


Fig. 6

(i) (A) Use the model to write down the mean of the maximum temperatures.

(B) Explain why the curve indicates that the standard deviation is approximately 3 degrees Celsius. **[2]**

Temperatures can be converted from Celsius to Fahrenheit using the formula $F = 1.8C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

(ii) For maximum temperature in October in degrees Fahrenheit, estimate

- the mean,
- the standard deviation. **[2]**

6	**(i)**	*A*	Mean = 17	**B1**	**3.4**	
		B	**Either** Points of inflection are approx. 3 above and below mean so SD = approx. 3	**E1**	**2.4**	AG
			Or Limits are approx. 9 above and below mean so SD = 9 ÷ 3 = 3	**E1**	**2.4**	AG
				[2]		

6	**(ii)**		Mean in Fahrenheit = 1.8 × 17 + 32 = 62.6 SD in Fahrenheit = 1.8 × 3 = 5.4	**B1** **B1** **[2]**	**1.1** **1.1**	FT their mean

7 Two events A and B are such that $P(A)=0.6$, $P(B)=0.5$ and $P(A\cup B)=0.85$. Find $P(A\mid B)$. **[4]**

7			$P(A\cap B) = P(A) + P(B) - P(A\cup B)$ $= 0.6 + 0.5 - 0.85$ $= 0.25$ $P(A\mid B) = \dfrac{P(A\cap B)}{P(B)}$ $= \dfrac{0.25}{0.5}$ $= 0.5$	**M1** **A1** **M1** **A1** **[4]**	**3.1a** **1.1** **1.1** **1.1**	

8 Alison selects 10 of her male friends. For each one she measures the distance between his eyes. The distances, measured in mm, are as follows:

51 57 58 59 61 64 64 65 67 68

The mean of these data is 61.4. The sample standard deviation is 5.232, correct to 3 decimal places.

One of the friends decides he does not want his measurement to be used. Alison replaces his measurement with the measurement from another male friend. This increases the mean to 62.0 and reduces the standard deviation. Give a possible value for the measurement which has been removed and find the measurement which has replaced it. **[3]**

8			Increases a value by 6 New value is closer to 62 than the old value is to 61.4 51 changes to 57 **or** 57 changes to 63 **or** 58 changes to 64	**M1** **M1** **A1** **[3]**	**3.1b** **2.2a** **2.2a**	Implied by correct answer or pair of values differing by 6 Implied by correct answer or new value closer to 62 than old value

9 A geyser is a hot spring which erupts from time to time. For two geysers, the duration of each eruption, x minutes, and the waiting time until the next eruption, y minutes, are recorded.

(i) For a random sample of 50 eruptions of the first geyser, the correlation coefficient between x and y is 0.758. The critical value for a 2-tailed hypothesis test for correlation at the 5% level is 0.279. Explain whether or not there is evidence of correlation in the population of eruptions. **[2]**

The scatter diagram in Fig. 9 shows the data from a random sample of 50 eruptions of the second geyser.

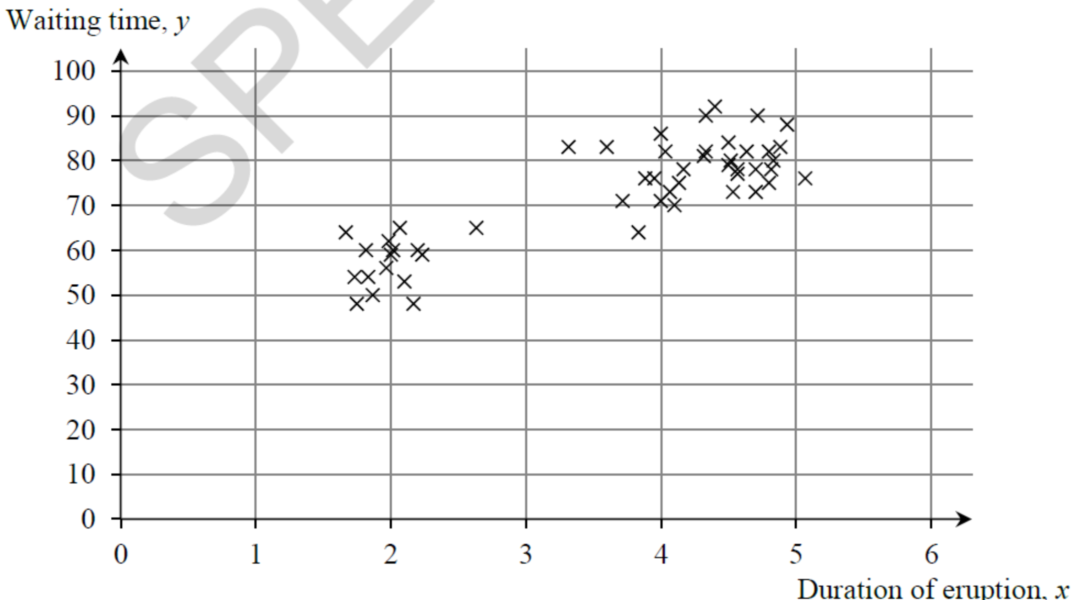


Fig. 9

(ii) Stella claims the scatter diagram shows evidence of correlation between duration of eruption and waiting time. Make two comments about Stella's claim. **[2]**

9	**(i)**		$0.758 > 0.279$ So there is sufficient evidence of correlation (in the population)	**M1** **A1** **[2]**	**1.1** **2.2b**	 Oe but not evidence of positive correlation.
9	**(ii)**		E.g. diagram shows positive correlation overall, but the data consists of two distinct clusters. E.g. neither of the two clusters show evidence of correlation	**B1** **B1** **[2]**	**2.3** **2.2b**	Accept other suitable correct comments

10 A researcher wants to find out how many adults in a large town use the internet at least once a week. The researcher has formulated a suitable question to ask. For each of the following methods of taking a sample of the adults in the town, give a reason why it may be biased.

Method A: Ask people walking along a particular street between 9 am and 5 pm on one Monday.

Method B: Put the question through every letter box in the town and ask people to send back answers.

Method C: Put the question on the local council website for people to answer online.

[3]

10	**(A)**		E.g. Will not sample people who work then/people who do not walk down that street.	**B1**	**2.4**	
	(B)		E.g. This will only get answers from those who want to send in an answer.	**B1**	**2.4**	
	(C)		E.g. This will only get answers from those who use the council website. E.g. Those who use the internet more frequently are more likely to see the question.	**B1**	**2.4**	
				[3]		

15 The quality control department of a battery manufacturing company checks the lifetimes of the batteries produced by the company. The lifetimes, x minutes, for a random sample of 80 'Superstrength' batteries are shown in the table below.

Lifetime	$160 \le x < 165$	$165 \le x < 168$	$168 \le x < 170$	$170 \le x < 172$	$172 \le x < 175$	$175 \le x < 180$
Frequency	5	14	20	21	16	4

(i) Estimate the proportion of these batteries which have a lifetime of at least 174.0 minutes. **[2]**

(ii) Use the data in the table to estimate

- the sample mean,
- the sample standard deviation. **[3]**

15	**(i)**		Estimated number $= 4 + \frac{16}{3} = 9\frac{1}{3}$	**M1**	**3.1b**	for attempt at interpolation
			$\frac{9\frac{1}{3}}{80} = 0.1166...$ so proportion is approximately 0.117	**A1**	**1.1**	
				[2]		
15	**(ii)**		E.g. Midpoints	**M1**	**1.1**	evidence of valid method for estimation
			Mean = 170	**A1**	**1.1**	**BC** Mean in the range 169-171
			Standard deviation = 3.4	**A1**	**1.1**	**BC** SD in the range 3-3.5
				[3]		

The data in the table on the previous page are represented in the following histogram:

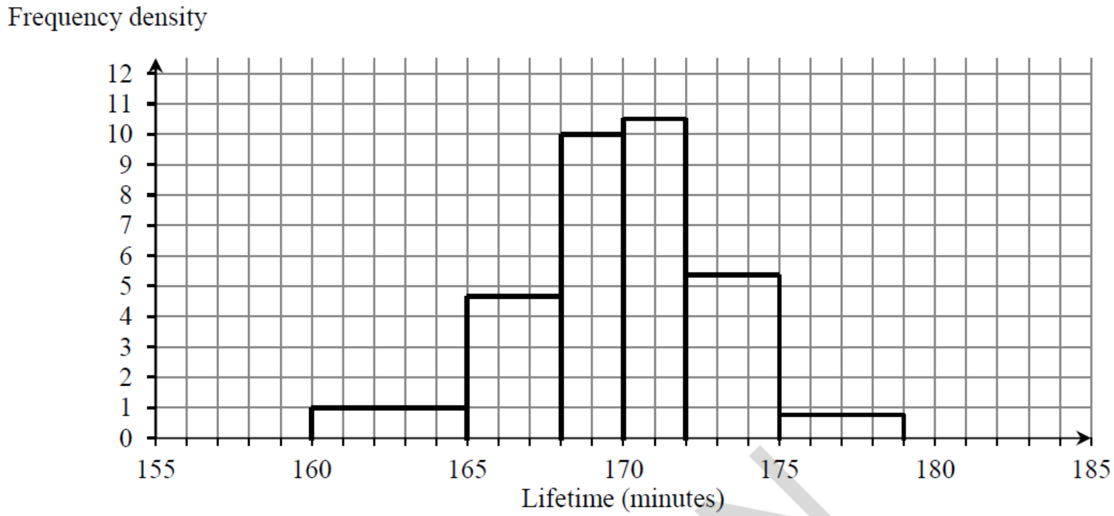


Fig. 15

A quality control manager models the data by a Normal distribution with the mean and standard deviation you calculated in part **(ii)**.

(iii) Comment briefly on whether the histogram supports this choice of model. **[2]**

(iv) (*A*) Use this model to estimate the probability that a randomly selected battery will have a lifetime of more than 174.0 minutes.

(*B*) Compare your answer with your answer to part **(i)**. **[3]**

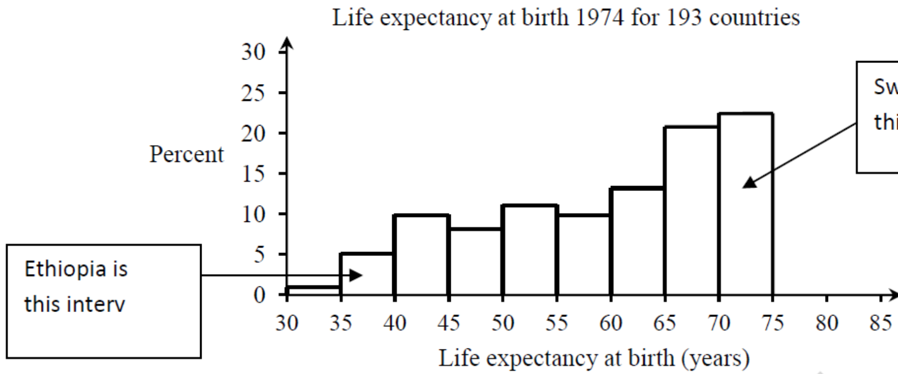
The company also manufactures 'Ultrapower' batteries, which are stated to have a mean lifetime of 210 minutes.

(v) A random sample of 8 Ultrapower batteries is selected. The mean lifetime of these batteries is 207.3 minutes. Carry out a hypothesis test at the 5% level to investigate whether the mean lifetime is as high as stated. You should use the following hypotheses $H_0: \mu = 210$, $H_1: \mu < 210$, where μ represents the population mean for Ultrapower batteries. You should assume that the population is Normally distributed with standard deviation 3.4. **[5]**

15	**(iii)**		The histogram e.g. seems to have a rough bell shape e.g. is symmetrical (around the estimated mean) e.g. appears to have all data within 3 s.d. of the mean	**B1**	**3.5a**	for one reason
			so this does support the manager's belief	**B1**	**3.5a**	for at least two reasons and 'supports belief'
				[2]		
15	**(iv)**	*A*	P(Lifetime > 174) for $N(170, 3.4^2)$	**M1**	**3.4**	oe
			0.1197	**A1**	**1.1**	**BC** FT their mean and standard deviation
		B	Answer is very similar to estimate in part (i)	**B1**	**3.5a**	
				[3]		

15	(v)		**Either** Test statistic $= \dfrac{207.3-210}{3.4/\sqrt{8}} = -2.246$	**M1**	**3.4**	Must include √8
			Lower 5% level 1 tailed critical value of $z = -1.645$	**A1**	**1.1**	
			$-2.246 < -1.645$ so significant	**B1**	**1.1**	For comparison leading to correct conclusion
			or Under H_0, $\overline{X} \sim N\left(210, \dfrac{3.4^2}{8}\right)$	**M1**	**3.4**	
			$P\left(\overline{X} \le 207.3\right) = 0.01235$	**A1**	**1.1**	**BC**
			$0.01235 < 0.05$ so significant	**B1**	**1.1**	
			There is sufficient evidence to reject H_0	**A1**	**2.2b**	
			There is sufficient evidence to conclude that the mean lifetime is less than 210 minutes.	**E1**	**2.4**	
				[5]		

16 Fig. 16.1, Fig. 16.2 and Fig. 16.3 show some data about life expectancy, including some from the pre-release data set.



A book

Fig. 16.1

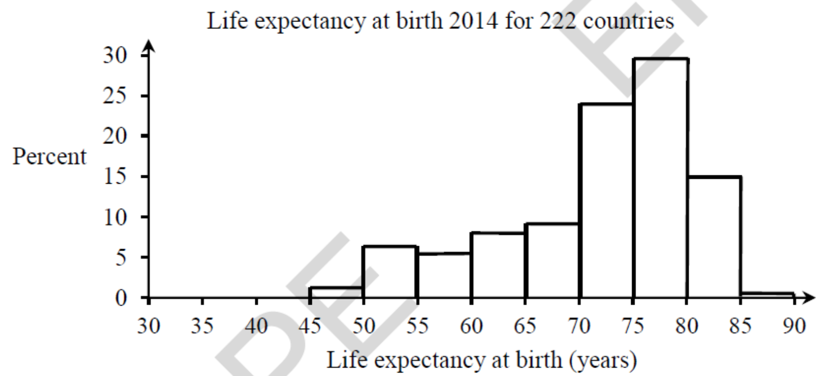


Fig. 16.2

Increase in life expectancy from 1974 to 2014 (years)

Increase in life expectancy for 193 countries from 1974 to 2014	
Number of values	193
Minimum	- 4.618
Lower quartile	6.9576
Median	9.986
Upper quartile	15.873
Maximum	30.742

Source: CIA World Factbook and Gapminder

Fig. 16.3

(i) Comment on the shapes of the distributions of life expectancy at birth in 2014 and 1974. **[2]**

(ii) (*A*) The minimum value shown in the box plot is negative. What does a negative value indicate? **[1]**

(*B*) What feature of Fig 16.3 suggests that a Normal distribution would not be an appropriate model for increase in life expectancy from one year to another year? **[1]**

(C) The values in the table in Fig. 16.3 have been obtained using software. Decide whether the level of accuracy is appropriate. Justify your answer. **[1]**

(D) John claims that for half the people in the world their life expectancy has improved by 10 years or more. Explain why Fig. 16.3 does not provide conclusive evidence for John's claim. **[1]**

(iii) Decide whether the maximum increase in life expectancy from 1974 to 2014 is an outlier. Justify your answer. **[3]**

Here is some further information from the pre-release data set.

Country	Life expectancy at birth in 2014
Ethiopia	60.8
Sweden	81.9

(iv) (*A*) Estimate the change in life expectancy at birth for Ethiopia between 1974 and 2014.

(*B*) Estimate the change in life expectancy at birth for Sweden between 1974 and 2014.

(*C*) Give **one** possible reason why the answers to parts (*A*) and (*B*) are so different. **[4]**

Fig.16.4 shows the relationship between life expectancy at birth in 2014 and 1974.

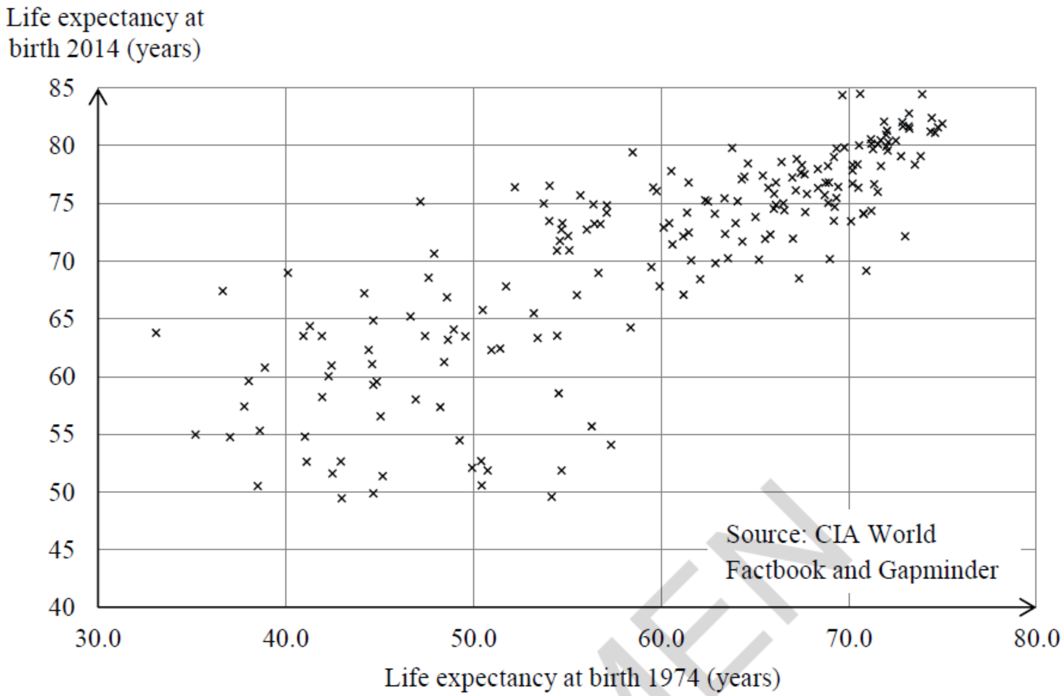


Fig. 16.4

A spreadsheet gives the following linear model for all the data in Fig 16.4.

(Life expectancy at birth 2014) = 30.98 + 0.67 × (Life expectancy at birth 1974)

The life expectancy at birth in 1974 for the region that now constitutes the country of South Sudan was 37.4 years. The value for this country in 2014 is not available.

(v) (*A*) Use the linear model to estimate the life expectancy at birth in 2014 for South Sudan. **[2]**

(*B*) Give two reasons why your answer to part (*A*) is not likely to be an accurate estimate for the life expectancy at birth in 2014 for South Sudan. You should refer to **both** information from Fig 16.4 **and** your knowledge of the large data set. **[2]**

(vi) In how many of the countries represented in Fig. 16.4 did life expectancy drop between 1974 and 2014? Justify your answer. **[3]**

Question			Answer	Marks	AOs	Guidance	
16	**(i)**		Comment about shape of distribution for first graph Comment about shape of distribution for second graph	**B1** **B1** **[2]**	**2.2b** **2.2b**	Comments can be combined e.g Both distributions negatively skewed gets both marks e.g. 1974 distribution has greater spread than 2014 gets both marks	If zero scored, **SC1** for "The 2014 distribution is shifted to the right of the 1974 distribution" oe
16	**(ii)**	***A***	Life expectancy went down [between 1974 and 2014] in [at least] one country	**E1** **[1]**	**2.2a**	**NOT** increase in life expectancy is negative	
	(ii)	***B***	The box plot is not symmetrical.	**B1**	**3.5b**		
				[1]			
	(ii)	***C***	Not appropriate with reason	**E1** **[1]**	**2.4**	e.g. [some] values of life expectancy are estimates The values of life expectancy are not available to this level of accuracy	
	(ii)	***D***	Comment about life expectancy at birth data for countries and not individual people	**B1** **[1]**	**2.4**		
16	**(iii)**		Use of $Q3 + 1.5 \times (Q3 - Q1)$ $15.873 + 1.5(8.9154) = 29.2461$ (years) The maximum value is an outlier as $30.742 > 29.2461$.	**M1** **M1** **A1** **[3]**	**1.2** **1.1** **1.1**		

Question			Answer	Marks	AOs	Guidance	
16	(iv)	*A*	 approx 60.8 - 37.5= 23.3 (years)	M1 A1	3.1b 1.1	Attempt to estimate change in life expectancy at birth soi. FT 'their 37.5 between 35 - 40'	
		B	Change in life expectancy for Sweden approx 81.9 - 72.5 = 9.4 (years)	 A1	1.1	 FT 'their 72.5 between 70 - 75'	
		C	E.g. Countries with a lower life expectancy in 1974 have greater opportunity to increase life expectancy in 2014.	E1 **[4]**	3.2a	OR Countries with a higher life expectancy in 1974 have less opportunity to increase life expectancy in 2014.	
16	(v)	A	$30.98 + 0.67 \times 37.4$ = 56.0 (years)	M1 A1 **[2]**	3.4 1.1		
		B	E.g. Large amount of scatter at the lower values [and South Sudan is 37.4]. E.g. Not having the data value could indicate that there are problems in the country which could mean it does not follow the pattern for other countries	E1 E1 **[2]**	3.5b 3.5b	E1 Reason inferred from Fig 16.4 E1 For knowing why data may be missing	
16	(vi)		Correct method Clearly explained 6	M1 E1 A1 **[3]**	3.1b 2.4 1.1	e.g. draw " $y = x$" on graph e.g. The value on the vertical axis must be lower than the one on the horizontal axis FT their correct method	

A level Mathematics:
support and next steps

Support

- We will be running subject knowledge courses:
  - Extended PD courses (TS, TM, TAM, TFM)
  - Face to face days
  - Online courses

- See www.furthermaths.org.uk/2017-cpd

Next Steps:

- What information was new?
- What implications are there for your students?
- What implications are there for your colleagues?
- What support will you need?
- Where can you get this support?

The Further Mathematics Support Programme

Our aim is to increase the uptake of AS and A level Further Mathematics to ensure that more students reach their potential in mathematics.

The FMSP works closely with school/college maths departments to provide professional development opportunities for teachers and maths promotion events for students.

To find out more please visit www.furthermaths.org.uk