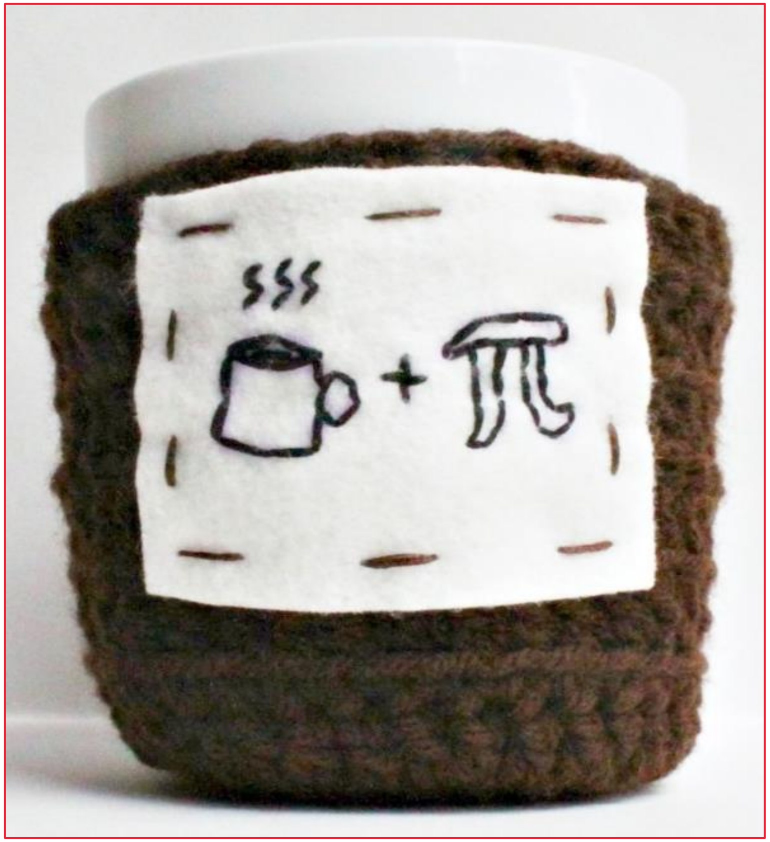
FMSP | Let maths take you further ®



Coffee and Pi

Mathematical argument, language and proof

with thanks to Elizabeth and Paul Glaister, article in ‘Mathematics in School’

Overarching Themes

- OT1 Mathematical argument, language and proof
- OT2 Mathematical problem solving
- OT3 Mathematical modelling

'must permeate the Content'

Mathematical argument, language and proof

Specifically, students must be encouraged to

- reason logically and recognize incorrect reasoning
- generalize mathematically
- construct mathematical proofs

'must permeate the Content'

Not a separate teaching topic

- Students should know and use correct mathematical language and argument across the whole of the Content.
- One way to develop this skill is to engage in formal proof.
- Keeping content simple will help students focus on the skills you are trying to develop.
- The skills need to be applied in all Content areas.

'must permeate the Content'

Four methods – are you confident with these?

- Proof by deduction
- Proof by exhaustion
- Disproof by counter-example
- Proof by contradiction

Proof by deduction

Prove that for all $n \in \mathbb{N}$, if n is odd, then n^2 is odd.

Proof by exhaustion

Prove that every integer that is a perfect cube is either a multiple of 9, or 1 less, or 1 more than a multiple of 9.

Disproof by counter-example

Disprove the statement that: for all $n \in \mathbb{Z}$,

the integer $f(n) = n^2 - n + 11$ is prime

Proof by contradiction

Prove that for all $n \in \mathbb{N}$, if $4^n - 1$ is prime, then n is odd.

And something for you…

2017 is a prime number, and $2017 = 9^2 + 44^2$.

Any prime number of the form $4n + 1$, where $n \in \mathbb{N}$, can always be expressed uniquely as the sum of squares of two natural numbers.

More generally, there is a theorem, originally written down by Fermat and subsequently proved by Euler, which states that all *odd* prime numbers p can be expressed uniquely as the sum of squares of two natural numbers $x, y \in \mathbb{N}$, i.e., $p = x^2 + y^2$, if an only if $p = 4n + 1$ for some $n \in \mathbb{N}$.

And something for you…

The last year for which this occurred was 1997 (= $29^2 + 34^2$), and this was also true for the previous 'prime year', 1993 (= $12^2 + 43^2$).

When is the next 'prime year' for which this is ture?

When is the next time two consecutive 'prime years' have this property?

[All this century, but some way off!]

When was the last time?

When is the next time that three 'prime years' in a row have this property?

[The last time the latter occurred was just after Fermat died, and the next time there are three successive occurrences of it. After this, the next time 'prime years' have this property there are five in a row. For four in a row, we have to go back to the time of the Magna Carta.]

The Further Mathematics Support Programme

Our aim is to increase the uptake of AS and A level Further Mathematics to ensure that more students reach their potential in mathematics.

The FMSP works closely with school/college maths departments to provide professional development opportunities for teachers and maths promotion events for students.

To find out more please visit www.furthermaths.org.uk