**Y13 A level Mathematics**

**35 Numerical Methods 1.5 weeks**

## Teaching objectives

**a Understand that some roots cannot be found using analytic methods; understand the importance of numerical methods**

**b Locate roots of functions by considering changes of sign, understanding how a change of sign method can fail**

**c Be able to carry out a fixed point iteration after rearranging an equation into the form x=g(x) and represent the process graphically (staircase and cobweb diagrams)**

**d Solve equations using the Newton-Raphson method and represent the process graphically**

**e Establish how iterative methods can fail to find a particular root**

**f Understand how to use numerical methods (specifically the trapezium rule) to find the approximate area under a curve and the limits that it must lie between**

**g Use numerical methods to solve problems in context**

**Resources for advance preparation:**

[**KM: True, sometimes, never**](http://www.kangaroomaths.com/free_resources/ks5/resources/trueneversometimes/c3numerical.ppt) **–** [**teacher notes**](http://www.kangaroomaths.com/free_resources/ks5/resources/trueneversometimes/teachernotesc3numerical.doc)

[**MEI: Investigating Iterative Formulae**](http://mei.org.uk/files/sow/34-numerical-methods-res.pdf)

[**UM: A cubic equation has one real root…**](https://undergroundmathematics.org/calculus-of-powers/r8231)- 1 sheet per pair

[**UM: Can we find an approximation to the root of this function?**](https://undergroundmathematics.org/geometry-of-equations/r5948)

[**UM: What is the area under the curve…?**](https://undergroundmathematics.org/calculus-trig-log/r8679)

[**GEO: Fixed point iteration**](https://www.geogebra.org/m/eXNx0wwG)

[**GEO: Trapezium rule**](https://www.geogebra.org/m/haGTeTk8)

[**NRICH: Root hunter**](http://nrich.maths.org/5876)

[**NRICH: Equation attack**](http://nrich.maths.org/5644)

[**NRICH: Two trees**](http://nrich.maths.org/298)

[**RISP39: Polynomial equations with integer coefficients**](http://www.s253053503.websitehome.co.uk/risps/risp39.html)

[**Mathsbox: Iterative formulae**](http://www.mathsbox.org.uk/resources/alevel/c3/Deriving%20Iterative%20formulae.pdf)

[**TES: Trapezium rule questions**](https://www.tes.com/teaching-resource/trapezium-rule-6146799)

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **1** | [**UM: A cubic equation has one real root…**](https://undergroundmathematics.org/calculus-of-powers/r8231)in pairs (this could be preceded by some equations for which students have analytic methods)During the starter, you may want to share (perhaps using one of the pairs’ work) the initial ideas of the bisection method for those pairs who haven’t found a way to begin. | Discussion of the starter should lead to:* The bisection method (change of sign) becoming secure (through sharing/demonstration)
* Discussion and understanding of the phrase ‘change of sign’ (What initial values should you choose? Why? How do you know if your initial values will be successful in finding a root?)
* Understanding of an error bound and ensuring that these are being stated (How accurate is your solution? What does it mean to state a value to a specified number of decimal places?)

Students can then continue with: [**NRICH: Root hunter**](http://nrich.maths.org/5876)Could add additional practice questions here to ensure students can perform the method correctly. | Extension: [**UM: Can we find an approximation to the root of this function?**](https://undergroundmathematics.org/geometry-of-equations/r5948) | Use plenary to discuss in more depth the failure of the change of sign method to locate roots |
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| **2** | How many rearrangements of the form x=g(x) can we find of y=f(x)?e.g:f(x) = quadraticf(x) = reciprocalf(x) = cubicetc.(for selection of functions, see next lesson too)At least one of the above functions to be solvable by known analytic methods (and don’t have trivial/integer roots) | Discuss solutions of the starter functions for f(x)=0, then get students to enter some pre-chosen values into g(x) that will converge. Ask the students to perform the next iteration and so on. Key questions are:* Why are these values converging on the solutions to f(x)=0? (focus here on the fact that the solutions to both equations are the same)
* Do they all converge to the nearest root?
* Is there anything else that you notice? (focus here on the variation between staircase and cobweb situations)
* What is happening graphically?

Students can then begin sketching the y=f(x), y=g(x) and y=x overlaid (could use graphing software for this).Discuss ideas arrived at from the sketches then students can then continue with (using geogebra):[**GEO: Fixed point iteration**](https://www.geogebra.org/m/eXNx0wwG)Discuss here the ideas of cobweb and staircase diagrams and ask the students to create, using the geogebra some examples of each (from what they have already done). | The functions that you use here should be recorded and used in the subsequent two lessons. | Plenary:Give some starting values for the functions that result in divergence. Ask the students to consider why this happens (but don’t answer until lesson 4!)Homework:[**Mathsbox: Iterative formulae**](http://www.mathsbox.org.uk/resources/alevel/c3/Deriving%20Iterative%20formulae.pdf) |
| **3** | Introduce the Newton-Raphson formula then ask students to differentiate the functions (f(x)) from the previous lesson’s starter/main activity | Ask students to choose sensible values (discussion point here) for input into the Newton-Raphson formulae for the functions then get the students to begin performing iterations. Key questions are:* Does the method converge to the nearest root/the one that you expected? Why/why not?
* What is happening graphically?
* Did the method ever fail? If so, why?
* Which converges quicker, Newton-Raphson or fixed-point iteration?
 | Extension: [**NRICH: Equation attack**](http://nrich.maths.org/5644) | Use some standard practice questions on iterative methods for homework |
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| **4** | Reminder of all of the points at which an iterative method failed to find a root – could use a spreadsheet or geogebra to show these numerically or graphically | Begin by asking the question:* When does fixed-point iteration fail to find a root?

Then:[**MEI: Investigating Iterative Formulae**](http://mei.org.uk/files/sow/34-numerical-methods-res.pdf)Follow this by giving some values for which the Newton-Raphson fails in the previous function.* Why does the Newton-Raphson fail to find a particular root?
* Are there any values for which the Newton-Raphson would fail to find any roots? Why does this happen?
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| **5** | Give the students a function that cannot be integrated by (their) known analytic methods.Ask the students in pairs to discuss possible ways in which the area could be approximated. | Discuss the ideas raised in the starter then introduce the trapezium rule. Students should then use:[**GEO: Trapezium rule**](https://www.geogebra.org/m/haGTeTk8)…to explore approximations to definite integrations of the starter function.Students should then complete the questions in the ‘notes and examples’ section from:[**TES: Trapezium rule questions**](https://www.tes.com/teaching-resource/trapezium-rule-6146799)Key questions:* When will the trapezium rule give an over-estimate/under estimate?
* How can a better estimate be obtained?
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| **6** | [**KM: True, sometimes, never**](http://www.kangaroomaths.com/free_resources/ks5/resources/trueneversometimes/c3numerical.ppt) | Use this lesson to solve some problems using the ideas built up over the past few lessons.[**RISP39: Polynomial equations with integer coefficients**](http://www.s253053503.websitehome.co.uk/risps/risp39.html) | [**NRICH: Two trees**](http://nrich.maths.org/298) | A good mixture of exams questions from integral (i.e. the assessments) or from a book would be appropriate homework here. |