**Y13 A2 Mathematics**

**34 Differential Equations 1.5 weeks**

## Teaching objectives

**a Be able to construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth and modelling the relationship between price and demand)**

**b Be able to evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions (Separation of variables may require factorisation involving a common factor)**

**c Be able to interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics**

**Resources for advance preparation:** See Consolidation column

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|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| 1 | Read[**The World Population**](http://mei.org.uk/files/papers/c4_june_2012.pdf#page=40)  A comprehension paper which involves modelling using differential equations | Explain that…for any continuously changing phenomena, in order to produce a mathematical model, the most effective starting point is usually to express the changes observed in terms of DEs. Being able to solve these DEs allows us, assuming the model is sufficiently accurate, to predict future behaviour of the system. Where an initial model is not sufficiently accurate, DEs provide an opportunity to explore the modelling cycle.  Explain that A level maths only entails a small subset of differential equations where the variables are separable.  i.e. DEs that can be expressed in the form  So we will be finding the general solution to equations like but not  Discuss the comprehension article from “The World Population” comprehension paper and define differential equations. Emphasise the need to decide between possible models and how you might decide. | Issues that may arise   * Difficulties rearranging expressions involving logarithms. For example,  and showing this is equivalent to * Errors interpreting negative rates of change and handling these in the formation of DEs in context questions * Dealing with the constant of integration where at least one term involves a logarithm.   Challenging questions  STEP 1 2011 Qu7  STEP 1 2013 Qu7  NRICH [Reaction rates](https://nrich.maths.org/6045) | MEI comprehension example [**The World Population**](http://mei.org.uk/files/papers/c4_june_2012.pdf#page=40)  Dr Frost A level 2017  [Integration](https://www.drfrostmaths.com/resource.php?rid=322) Slides 60-69  [RISP 30](http://www.s253053503.websitehome.co.uk/risps/risp30.html) How many differential equations - gives practice of solving differential equations  [RISP 28](http://www.s253053503.websitehome.co.uk/risps/risp28.html) Modelling the spread of disease gives an example of how a DE can model a real situation  Solomon Worksheets  [Worksheet K](https://pmt.physicsandmathstutor.com/download/Maths/A-level/C4/Worksheets-Notes/Solomon/C4%20Integration%20K%20-%20Questions.pdf) [(K Ans)](https://pmt.physicsandmathstutor.com/download/Maths/A-level/C4/Worksheets-Notes/Solomon/C4%20Integration%20K%20-%20Answers.pdf)  [Worksheet L](https://pmt.physicsandmathstutor.com/download/Maths/A-level/C4/Worksheets-Notes/Solomon/C4%20Integration%20L%20-%20Questions.pdf) [(L Ans)](https://pmt.physicsandmathstutor.com/download/Maths/A-level/C4/Worksheets-Notes/Solomon/C4%20Integration%20L%20-%20Answers.pdf)  Madasmaths booklets  [Worded differential equations](https://www.madasmaths.com/archive/maths_booklets/standard_topics/integration/odes_context_modelling.pdf)  [More differential equations](https://www.madasmaths.com/archive/maths_booklets/standard_topics/integration/odes_separable_no_context.pdf)  Colmanweb Steps to solving DEs [DE writing frame](http://www.colmanweb.co.uk/Assets/Resources/DiffEqnWritingFrame.pdf) |
| 2 | Find three different solutions to the equation  Solve  Given that  when | In relation to the starter question define the terms ‘general solution’ and ‘particular solution’.  Show that you could separate the variables and are essentially then integrating both sides of these equations:  Use the second example to check that students can manipulate logarithms and to show that they only need to add a constant to one side of the general solution because if they add one on each side they can just be reduced to a single constant.  Explain that they can separate the variables so that the left hand side is not just a 1 if the right hand side involves both variables. Show why this works when the right hand side is xy but not when it is x+y.  Practice a few using [RISP 30](http://www.s253053503.websitehome.co.uk/risps/risp30.html) |
| 3 | Recap rearranging exponential equations using a few questions from Solomon worksheets  [C3 Sheet A](https://pmt.physicsandmathstutor.com/download/Maths/A-level/C3/Worksheets-Notes/Solomon/C3%20Exponentials%20and%20Logarithms%20A%20-%20Questions.pdf)  [C3 Sheet B](https://pmt.physicsandmathstutor.com/download/Maths/A-level/C3/Worksheets-Notes/Solomon/C3%20Exponentials%20and%20Logarithms%20B%20-%20Questions.pdf) | Model how to deal with logarithms on both sides of the solution.  Practise solving DEs using Colman Steps to solving DEs [DE writing frame](http://www.colmanweb.co.uk/Assets/Resources/DiffEqnWritingFrame.pdf) |
| 4 | NRICH [Differential equation match](https://nrich.maths.org/5875) | Discuss the clues in this worded question that allow you to create a DE  A spherical balloon which is being inflated has radius r cm at time t seconds. It takes 3 seconds to inflate the balloon to a radius of 16 cm from its initial value of 1 cm. In a simple model, the rate of increase of r is taken to be proportional to . Express this as a differential equation and find the time it would take to inflate the balloon to a radius of 20 cm, giving your answer in seconds to 2 significant figures.  Practise writing and solving DEs using Solomon worksheets K and L |
| 5 | Try to write a DE for this example:  According to Newton’s Law of Cooling, the rate at which the temperature of a body falls is proportional to the amount by which the temperature exceeds that of its surroundings. A room is at a constant temperature of . An object has temperature  when it is brought into the room, and 5 minutes later its temperature is . What will its temperature be after a further 5 minutes? | Discuss starter. Emphasise the need to consider whether negatives are needed, and to be clear about defining variables. Look at [RISP 28](http://www.s253053503.websitehome.co.uk/risps/risp28.html) together.  Practise writing and solving DEs from worded problems using Solomon or Madasmaths resources. |
| 6 | Try to write a DE for this example:  A mathematical model for the number of bacteria, n, in a culture, states that n is increasing at a rate proportional to the number present. At 11:00am there are 500 bacteria and at 11:30am there are 735. At what time will there be 1000 bacteria? |  |