**Y13 A level Mathematics**

**31 Integration 3.5 weeks**

## Teaching objectives

**a To review knowledge of integration from Y12**

**b To be able to integrate expressions by inspection using the reverse of the chain rule**

**c To be able to integrate ekx and lnx**

**d To be able to integrate trigonometric functions using trig identities**

**e Use integration by inspection**

**f Integrate using partial fractions with a linear denominator and simplify the result**

**g Integrate using substitution including selecting the correct substitution**

**h Integration by parts**

**I Use integration as a limit of a sum**

**J Find the area between 2 curves including choosing the appropriate method of integration**

**Resources for advance preparation:**

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|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **1** | UM: can we find the area inside a parabola, a tangent and the x-axis?  <https://undergroundmathematics.org/calculus-of-powers/r6584>  (extended starter) | Discuss different methods available to find area under curve  Recap basics of definite and indefinite integration including relevance of +c  Ensure students understand that integration can be used as reverse of differentiation and to find area under curve | Consider pairings, match strong students with weak. ALL students should be expected to feedback and demonstrate working | Old exam question on finding equation of line given dy/dx and a point the equation passes through  HOMEWORK:  Routine practice, particularly of finding equations of lines and areas under curves |
| **2** | Mini whiteboards  Students to create a polynomial function that they then differentiate. Hold up differentiated function, class to determine original function. | Develop understanding of use of chain rule to aid method of integration by inspection. E.g. ,  Dr Frost maths – integration powerpoint  <http://www.drfrostmaths.com/resources/resource.php?rid=63> | students often forget to divide by the differential of the base function, so it is important to ensure all students have a clear understanding of this process. | HOMEWORK:  Routine practice from old resources |
|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **3** |  | **Introduce integration of ex and ln x, link to general principles of integration**  **UM: Two for one**  [**https://undergroundmathematics.org/calculus-trig-log/two-for-one**](https://undergroundmathematics.org/calculus-trig-log/two-for-one)  Dr Frost maths – integration powerpoint  <http://www.drfrostmaths.com/resources/resource.php?rid=63> |  | HOMEWORK:  Routine practice from old resources |
| **4** | UM: trigsy integrals  <https://undergroundmathematics.org/calculus-trig-log/trigsy-integrals> | Investigation into integration of basic trig functions, using reverse of differentiation.  Use knowledge of trig differentiation to find standard results  Dr Frost maths – integration powerpoint  <http://www.drfrostmaths.com/resources/resource.php?rid=63> |  | UM: Where do the curves y = sin 2x and y = sin x cross?  <https://undergroundmathematics.org/calculus-trig-log/r7074>  HOMEWORK:  Routine practice from old resources  <https://www.madasmaths.com/archive/maths_booklets/standard_topics/integration/integration_by_trigonometric_identities.pdf> |
| **5/6** |  | Use trig identities to integrate more complex functions.  Use of double angle formulae, basic identities required  e.g. tan2x +1 = sec2x  Dr Frost maths – integration powerpoint  <http://www.drfrostmaths.com/resources/resource.php?rid=63>  <https://www.teachitmaths.co.uk/resources/ks5/integration/integrating-trigonometric-functions/24064> | [**https://www.tes.com/teaching-resource/further-trig-integration-6146919**](https://www.tes.com/teaching-resource/further-trig-integration-6146919) |  |
|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **7** |  | Integration by inspection e.g. cosxsin2x,  Tie in with previous 6 lessons  <https://www.tes.com/teaching-resource/parts-substitution-recognition-6146851>  <https://www.tes.com/teaching-resource/hard-integration-by-recognition-matching-activity-6322946> |  | HOMEWORK:  Routine practice from old resources |
| **8/9** | Revision of partial fractions may be useful | **Partial fractions,**  Including definite integration and simplification of answers using ln  [**https://www.tes.com/teaching-resource/a-level-maths-integration-worksheets-6146916**](https://www.tes.com/teaching-resource/a-level-maths-integration-worksheets-6146916)  [**https://www.strath.ac.uk/media/other/mathsskills/resources/level1materials/Partial\_fractions.pdf**](https://www.strath.ac.uk/media/other/mathsskills/resources/level1materials/Partial_fractions.pdf) | Recap of simplifying expressions using laws of logs  Extension: repeated factors in the denominator | HOMEWORK:  Routine practice from old resources  <https://www.madasmaths.com/archive/maths_booklets/standard_topics/integration/integration_partial_fractions_student_version.pdf> |
| **10/11** |  | **Introduction of integration by substitution**  <https://www.tes.com/teaching-resource/integration-by-substitution-worksheet-6152845> |  | HOMEWORK:  Routine practice from old resources |
| **12/13** |  | **Integration by substitution, selection of appropriate substitution.**  **Link to areas under curves and problem solving**  <https://undergroundmathematics.org/chain-rule/integral-sorting> useful for practising selecting the appropriate substitution  <https://undergroundmathematics.org/chain-rule/which-substitution> |  | HOMEWORK:  Routine practice from old resources |
|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **14/15** | [**https://integralmaths.org/pluginfile.php/9150/mod\_page/content/5/Teach\_Int\_Methods.pdf**](https://integralmaths.org/pluginfile.php/9150/mod_page/content/5/Teach_Int_Methods.pdf)  (second lesson)  [**https://undergroundmathematics.org/product-rule/integral-chasing-2**](https://undergroundmathematics.org/product-rule/integral-chasing-2) | Integration by parts  Derivation of formula and appropriate use  Discussion as to choice of u and dv. Why is it best to make the polynomial u? When will this not work?  Particular care of ln x and use of formula twice  Look at examples like using by parts and substitution to show they give the same answer.  [**h**ttps://www.tes.com/teaching-resource/maths-ks5-core-4-integration-by-parts-worksheets-6095737](https://www.tes.com/teaching-resource/maths-ks5-core-4-integration-by-parts-worksheets-6095737) | A useful flow chart to assist with selecting appropriate method of integration  <https://www.tes.com/teaching-resource/a-level-maths-integration-worksheets-6146916>  <https://undergroundmathematics.org/product-rule/r8134>  problem solving: consider integral of excos x  common errors- incorrect signs when using parts twice | HOMEWORK:  Routine practice from old resources  <https://www.madasmaths.com/archive/maths_booklets/standard_topics/integration/integration_by_parts_student_version.pdf>  <https://www.tes.com/teaching-resource/integration-by-parts-worksheet-6152850> |
| **16** | <https://www.stem.org.uk/resources/elibrary/resource/35717/integration> | **Area between 2 curves**  [**https://undergroundmathematics.org/chain-rule/slippery-areas**](https://undergroundmathematics.org/chain-rule/slippery-areas)  **Can we find the area between sin x and sin 2x?**  [**https://undergroundmathematics.org/calculus-trig-log/r9184**](https://undergroundmathematics.org/calculus-trig-log/r9184) |  | HOMEWORK:  Routine practice from old resources |
|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **17** | <https://undergroundmathematics.org/product-rule/constant-consistency> | Appropriate methods  Use of ICT  <https://undergroundmathematics.org/product-rule/r8152> | [**https://www.tes.com/teaching-resource/a-level-maths-integration-worksheets-6146916**](https://www.tes.com/teaching-resource/a-level-maths-integration-worksheets-6146916) | HOMEWORK:  Dr frost maths: compilation of edexcel old exam questions  <http://www.drfrostmaths.com/resources/resource.php?rid=63>  <https://www.madasmaths.com/archive/maths_booklets/standard_topics/integration/integration_structured_exam_questions_part_i.pdf> |
| **18** |  | **Understand and use integration as the limit of a sum?????**  [**https://undergroundmathematics.org/sequences/sum-estimating**](https://undergroundmathematics.org/sequences/sum-estimating)  **Links to trapezium rule & fundamental theorem of calculus** |  | HOMEWORK:  Routine practice from old resources |