**Y13 A level Mathematics**

**28 Algebra 2 weeks**

## Teaching objectives

**a Review expanding, collecting, factorisation from AS**

**b Simplify rational expressions using factorising and cancelling**

**c Review algebraic division and factor theorem**

**d Extend algebraic division**

**e Introduce partial fractions**

**f More complicated partial fractions**

**g Review binomial expansion for positive integer**

**h Extend binomial expansion to any rational n and understand validity**

**i Use of partial fractions with binomial expansion**

**j Other Uses of partial fractions eg integration, series**

**Resources for advance preparation:**

Old edexcel textbook references: Core 3 Chapter 1 (algebraic fractions) and Core 4 Chapter 1 (partial fractions) and 3 (binomial expansion).

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|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **1** | <https://undergroundmathematics.org/polynomials/divide-it-up>  This starter connects multiplication and factorising with division | **Review expanding and factorisation from AS**  **(if needed)**  Follow up starter to highlight any misconceptions or problems  Give students mini whiteboards and put up questions to check basic concepts   * expand two and three brackets * expand brackets with more than two terms in them * factorise quadratics and simple cubics * recognise difference of two squares * factorise quadratics with coefficient more than one * complete the square | Follow up problem areas with consolidation exercises from your old Core 1 textbook. | <http://www.transum.org/software/SW/Starter_of_the_day/Students/Factorising.asp?Level=6>  Self marking factorising online tool!  Key Question for Plenary: How does factorising and completing the square enable you to sketch the graph of a quadratic function? |

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| **2** | If you didn’t do lesson 1 then use this starter now:  <https://undergroundmathematics.org/polynomials/divide-it-up>  This starter connects multiplication and factorising with division  Otherwise:  Give students mini whiteboards:  Put up cancelling-type problems including numerical fractions eg 2/3 \* ¾\* 4/5\*5/6 etc and the classic algebra misconceptions, eg (2+x)/(1+x)  Plus some examples which will cancel if they factorise  Eg (2x + x2)/(x3 – 3x)  Also some with fractions in numerator and/or denominator | **Simplify rational expressions using factorising and cancelling**  Go through particular techniques which can be helpful when trying to simplify an algebraic fraction – ie factorising or multiplying through top and bottom by a number or algebra. Give examples for students to practise.  Ask students to write down the four operations and how they deal with them with numerical fractions.  Then ask them to discuss what is the same or different with algebraic fractions.  Give examples of adding, subtracting, multiplying and dividing.  This RISP shows how to use Arithmagons to help with doing and undoing algebra.  <http://www.s253053503.websitehome.co.uk/risps/risp21.html>  Use figures 9 and 10. | Extend - question on addition and subtraction which require factorising first. <https://undergroundmathematics.org/thinking-about-algebra/r5364> | [**http://www.transum.org/software/SW/Starter\_of\_the\_day/Students/Algebraic\_Fractions.asp?Level=5**](http://www.transum.org/software/SW/Starter_of_the_day/Students/Algebraic_Fractions.asp?Level=5)  **Self marking algebra cancellation online tool**  Consolidation: Use old Core 3 textbook or MEI integral exercise Level 1  Plenary:  Ask students to discuss common errors which they may make when dealing with algebraic fractions and document these as a class. |

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| **3** | <https://undergroundmathematics.org/polynomials/whats-left>  This starter looks at remainders when you divide polynomials | **Review Algebraic Division and Factor Theorem from AS**  **(if needed)**  Choose a few of the division problems from the starter and ask students to do the division using both the (reverse multiplication) grid method and long division.  Now give a few problems where there is no remainder for them to work through.  See if they remember the factor theorem and work through what it means from both directions ie, if f(p) = 0 then (x-p) is a factor, and if (x-p) is a factor, then f(p) = 0  Extend to remainder theorem (not sure if this is still on the syllabus but I would do it anyway) and compare a long division method giving remainder, with using the remainder theorem for an example with a linear denominator. | Extension:  <https://undergroundmathematics.org/polynomials/r6577> | Follow up problem areas with consolidation exercises from your old Core 2 textbook or MEI integral exercise Level 1  Plenary:  <https://undergroundmathematics.org/polynomials/r6488> |
| **4** | If not used in lesson 3, use this starter now:  <https://undergroundmathematics.org/polynomials/whats-left>  This starter looks at remainders when you divide polynomials | **Extend algebraic division**  Ask students to give examples of top heavy fractions, they will start with numeric examples but get them to think about what top heavy means for an algebraic fraction – leading to looking at the highest power (degree) of numerator vs denominator.  From the starter form an idea of how the degree of the answer (quotient) and the degree of the remainder relates to the relative degree of the numerator and denominator. Answer – if a polynomial of degree n is divided by a polynomial of degree r then the quotient has degree at the most n – r and the remainder has degree at most r-1  Give examples for students to work through with top heavy algebraic division using one or more of three methods: long divison, equating coefficients (ie remainder theorem) or inspection. | **Extension:**  <https://undergroundmathematics.org/polynomials/r6488> | Follow up problem areas with consolidation exercises from your old Core 3 textbook or MEI integral exercise Level 1 or 2  Plenary:  Use division to help sketch a graph of  y = (2x – 5)/(x – 3) |
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| **5** | MEI Casio/Desmos/Geogreba tasks A2 Core 9  <http://mei.org.uk/integrating-technology>  Go down list to **Algebra** topic in A2 (not AS) to find the tasks.  Students use their graphical calculators or graphing packages to adjust coefficients to make graphs the same giving an introduction to the equivalence of partial fractions | **Introduce partial fractions**  Students work in pairs to algebraically show the equivalence of the first set of graphs plotted in the starter.  Students to demonstrate solutions.  Teacher leads students through a method to find coefficients, if this has not come out already or brings together methods. Make sure you cover both equating coefficients and substitution.  Try the method(s) out with the second example in the starter, then plot to check equivalence.  Show an example where the denominator is not factorised and ask students to discuss what they would do to convert to partial fractions. | This may throw up problems with using common denominators in algebra so support by going through some examples of adding fractions with different levels of complexity in denominators.  Extend by looking at the further examples on the starter.  **Extension:**  [**https://undergroundmathematics.org/polynomials/frightening-function**](https://undergroundmathematics.org/polynomials/frightening-function) | Consolidation:  Use exercises in old C4 textbook or MEI exercise level 1  If you subscribe to MEI there is an Always, Sometimes, Never Task which you could use as a plenary.  Key Question:  Why could it be useful to split an algebraic fraction into partial fractions? |

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| **6** | Put up example of algebraic fraction with repeated factors and ask students to experiment with splitting into partial fractions. | **More complicated partial fractions - repeated factors or top heavy fractions**  Most students will miss out all the partial fractions needed and will come up with coefficients which do not work with each other. If no students have come up with all factors needed, go through this.  Do an example together with a repeated factor.  Put up an example of a top heavy algebraic fraction with factorised denominator and ask students to discuss how to approach this. Some may remember from the previous lesson about remainders and either try to do long division or predict what the answer would look like and match coefficients. Go through both approaches as a class and give practise questions. | **Extension: Use same extension as last lesson.** | Use exercises in old C4 textbook or MEI exercise level 2  Plenary:  Kangaroo Maths Free Resource – powerpoint with statements to be judged True, Never, Sometimes – select the C4AlgebraPPT from this screen:  <http://www.kangaroomaths.com/free_resources/ks5/resources/trueneversometimes/> |
| **7** | Use graph plotting software to look at the equivalence of eg y=(1+x)^3 and building up the graph from the expansion components ie y = 1 + 3x + …. | **Review Binomial Expansion for positive integer**  **(lesson may or may not be necessary or could be combined with lesson 8)**  Give out mini whiteboards:  First check knowledge of factorial and combination notation (with and without calculator)  Then put binomial expansion in formula book up on screen:  Give examples on binomial expansion with positive integer for students to try. Include where coefficient of “x” is negative and non unity.  Students explore using the binomial expansion to work out numerical values using:  [**https://undergroundmathematics.org/counting-and-binomials/r7978**](https://undergroundmathematics.org/counting-and-binomials/r7978)  Remind students how to work out a single term in a binomial expansion and then have quick-(ish) fire with mini whiteboards, for them to find a particular term in an expansion: eg the x3 term in the expansion (3 – 2x)5 | **Extension:**  Here is a problem which will bring out misconceptions where the coefficient in the binomial expansion is a fraction  [**https://undergroundmathematics.org/counting-and-binomials/r5563**](https://undergroundmathematics.org/counting-and-binomials/r5563) | Plenary:  Include examples where the binomial expansion is multiplied by another expression and ask for a particular term within that expansion.  Eg x3 term in (1+x)4(2x2 – 5x + 3)  Key Question:  What does the C in nCr stand for and how does that calculation actually work in the binomial expansion (or it is just magic?) |
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| **8** | Ask students to work out how many terms (without doing the expansion) that you would end up for different examples of binomial expansion with a positive integer.  Eg How many terms in (1+x)5 ?  (2-3x)7  (a+b)21 | **Extend binomial expansion to any rational n and understand validity**  Clarify that when using binomial expansion with a positive integer there is a fixed number of terms as the powers count down to zero.  Introduce to students the idea that the power in a binomial expansion could be a fraction or negative and the binomial expansion formula still holds true (as long as the series converges – explained next)  Now as the power counting down will never get to zero, this means there are an infinite number of terms. This means that “x” must be less than 1 otherwise the series would just get bigger and bigger.  Go through a couple of examples using the binomial theorem (keep the number as 1 at this stage), one with negative power and one with fractional power, emphasising the usual misconceptions, (which are that students find it conceptually hard to subtract 1 from the power when it is fractional or negative, and if the coefficient of x is not unity they forget to power it). Make sure you talk about the validity range of x for every example.  Students practise some examples, make sure they cover ones where the “x” has a coefficient which is non unity.  Ask students to work out how to use an expansion with x as a specific value to find a decimal approximation to a square root. Eg in (1-3x)^0.5 to find out approximation to sqrt(97). Highlight how accuracy increases the more terms you use. | **If you have MEI login there is a Binomial Tarsia which you could use for consolidation**  **Extension: RISP 22:**  [**http://www.s253053503.websitehome.co.uk/risps/risp-22.pdf**](http://www.s253053503.websitehome.co.uk/risps/risp-22.pdf)  **or MEI exercise Level 3** | Homework - consolidation exercises from your old Core 4 textbook or MEI integral exercise Level 1 or 2  Plenary: Ask students to discuss key errors they could make when expanding a binomial with non positive integer power, and document these as a class. |

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| **9** | This free MEI resource contains many of the common errors made when using the binomial theorem – students mark the answers and correct mistakes.  [**http://mei.org.uk/files/sow/28-algebra-res.pdf**](http://mei.org.uk/files/sow/28-algebra-res.pdf) | **More complicated binomial expansions Part 1 – taking out a common factor, combining expansions and making approximations.**  Follow up any misconceptions highlighted by the starter.  Introduce an example of (a + bx) ^ n with a being greater than 1 and show how to take out the number as a factor.  Introduce an example with a multiplication and then division. Ask students to discuss the validity where there is a combination of expansions.  If not done last lesson, Introduce an example with a numerical approximation and show how the accuracy increases the more terms you use. | **Extension: RISP 19:**  [**http://www.s253053503.websitehome.co.uk/risps/risp19.html**](http://www.s253053503.websitehome.co.uk/risps/risp19.html)  **Extension on numerical approximation:**  [**https://undergroundmathematics.org/counting-and-binomials/r6503**](https://undergroundmathematics.org/counting-and-binomials/r6503) | Homework - consolidation exercises from your old Core 4 textbook or MEI integral exercise Level 1 or 2  Plenary:  Kangaroo Maths Free Resource – powerpoint with statements to be judged True, Never, Sometimes – select the C4BinomialPPT from this screen:  <http://www.kangaroomaths.com/free_resources/ks5/resources/trueneversometimes/> |
| **10** | **Mini whiteboards-**  A partial fraction starter to remind them how to tackle these. | **More complicated binomial expansion Part 2 - Use partial fractions with expansion**  Ask the students to work out how you could use partial fractions to expand a polynomial fraction. How do you decide how many terms to expand? How do you work out the range of validity for x? |  | Homework - consolidation exercises from your old Core 4 textbook or MEI integral exercise Level 1 or 2  Assessment: Use a test from MEI or old C3/C4 exam questions to test algebra knowledge. |
| **11** | [**https://undergroundmathematics.org/polynomials/r6317**](https://undergroundmathematics.org/polynomials/r6317)  This shows how division can be used to sketch a graph | **Extension lesson - Other uses of division and partial fractions – integration and infinite series**  If integration has been covered, take students through how to use partial fractions to integrate certain expressions.  Show how partial fractions can be used to evaluate some infinite series.  This is an example using partial fractions to evaluate a finite series:  [**https://undergroundmathematics.org/counting-and-binomials/r5245**](https://undergroundmathematics.org/counting-and-binomials/r5245) |  |  |