**Y13 A level Mathematics**

**25 Functions 2.5 weeks**

## Teaching objectives

**a To review knowledge of functions from Year 12**

**b To understand and use the modulus of a linear function**

**c To understand and use composite functions**

**d To understand and use inverse functions and their graphs**

**e To use functions in modelling, including consideration of limitations and refinements of the models.**

**Resources for advance preparation:**

**Integral Maths (old spec) – Transforming graphs of functions, Modulus Function: Activity 2 worksheet:** 1 per pair.

**Integral Maths (old spec): Functions: Composite and Inverse functions: Activity 5:** 1 per pair worksheet questions

**Integral Maths (old spec): Functions: Composite and Inverse functions: Activity 7:** 1 copy per student (write on worksheet)

**Integral Maths (old spec): Functions: Composite and Inverse functions: Activity 8:** 1 copy per student (jigsaw)

[**Worksheet**](http://cbschoolshs.sharpschool.net/UserFiles/Servers/Server_5219792/File/Boulton/Classwork%20Inverses.pdf)– Question Sheet: 1 per student

[**Website questions 1**](http://www.purplemath.com/modules/fcncomp5.htm)⎞Create a worksheet

[**Website questions 2**](http://www.analyzemath.com/CompositionFunction/applications_composition.html)⎬ of questions from these sources

[**UM: Compose**](https://undergroundmathematics.org/combining-functions/compose)– Question Sheet: 1 per pair (or could be projected)

[**UM: Piece it together**](https://undergroundmathematics.org/thinking-about-functions/piece-it-together) – Question cards: 1 per pair (from problem sheet)

 – Equation Cards: 1 sheet per pair

[**UM: Sketch and describe composite functions**](https://undergroundmathematics.org/combining-functions/r5595/solution)

– Question and Suggestion sheets: 1 per Pair

[**UM: Function Squares**](https://undergroundmathematics.org/thinking-about-functions/function-squares/main-problem)– Question cards: 1 per pair

 – Functions cards: 1 per pair (double sided)

[**TES: Functions Murder Mystery:**](https://www.tes.com/teaching-resource/functions-murder-mystery-6437381) – Mystery sheets: 1 per pair

[**TES: Inverse dominoes**](https://www.tes.com/teaching-resource/core-3-inverses-dominoes-game-6033007)**:** – Domino Cards: 1 per pair

[**TES: Composite Functions**](https://www.tes.com/teaching-resource/c3-composite-functions-introductory-activity-11112493)**:** – Write on worksheet: 1 per student or pairs

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **1 & 2** | [**UM: Piece it together**](https://undergroundmathematics.org/thinking-about-functions/piece-it-together)In pairs students are introduced to the idea that graphs can take a piecewise form rather than being continuous. | Introduce the Modulus graph using [**UM: Absolutely**](https://undergroundmathematics.org/thinking-about-functions/absolutely)Discuss as a group the graph and questionsGraph sketching: use a combination of drawing and reflecting the original graph and the application of transformations to the modulus graph y = |x|.**Integral Maths (old spec): Transforming graphs of functions: Modulus function: Activity 2:** Graphs and equation matching activity. | Consider whether you can tell if the graph is of the form y = |f(x)| or y = f(|x|) from this graph. When could you tell?This could be investigated using Geogebraor[**CASIO CG20 Task**](http://mei.org.uk/casio-networks)**:** use the activity under: Full A Level; Functions; The Modulus Function (Task 1) | [**Khan Academy**](https://www.khanacademy.org/math/algebra/absolute-value-equations-functions)**: Absolute value and piecewise functions activities**Select students to come and solve each of the questions in the practice sections. They should talk the class through their processes.**Homework:**Exam questions on sketching modulus graphs |
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| **3** | [**UM: Can we sketch the graph of y=|x|+|x−1|+|x−4|?**](https://undergroundmathematics.org/thinking-about-functions/r9130)Review of Piecewise functions and the modulus graph. Here you are encouraging the students to consider whether you are using y = f(x) ory = -f(x) in each region | Solving modulus equations:Discuss possible methods for solving equations: Algebraic/graphical. Consider which graphs could be used for solving |x+2| = 3Use the graph to determine an algebraic method for solving the equation considering using y = f(x) and y = -f(x). Web questions, examples and solutions [**Maths Warehouse Activity**](http://www.mathwarehouse.com/absolute-value/how-to-solve-absolute-value-equation.php#practiceProblems1) | Ask whether 2 graphs are necessary or just desirable.Encourage students to recognise whether solutions are on reflected sections of the graph or not. | [**UM: solving inequalities, = 1/x**](https://undergroundmathematics.org/thinking-about-functions/r8079)**Homework:**Exam questions on solving modulus equations |
| **4, 5** **& 6** | [**UM: Domain and range dominoes**](https://undergroundmathematics.org/combining-functions/domain-and-range-dominoes)Refresh students’ memories about the range and domain of functions. | Discuss evaluating successive functions, are the same results achieved in any order?Introduce notation gf(x) = g ○ f(x) = g(f(x)) Use Composing Functions section of Khan Academy either as a group activity or as an individual activity[**Khan Academy: Composite functions:**](https://www.khanacademy.org/math/algebra2/manipulating-functions)**Progression:** Consider the range of composite functions and deconstructing composite functions: Pair task: [**UM: Compose**](https://undergroundmathematics.org/combining-functions/compose)[**TES: Composite Functions**](https://www.tes.com/teaching-resource/c3-composite-functions-introductory-activity-11112493)**:** Finding components given varying pairs of information **Integral Maths (old spec): Functions: Composite and Inverse functions: Activity 5:** Deconstructing composite functions**Extension:** [**UM: Sketch and describe composite functions**](https://undergroundmathematics.org/combining-functions/r5595/solution) | Establish that the order of the functions matters.Composing functions has 3 sections: Evaluating functions; Evaluating using tables and graphs; creating composite functions.Extending the interpretation of composite functions in order to draw complex graphs (this activity includes exponential functions). | [**UM: Composing gets me nowhere**](https://undergroundmathematics.org/combining-functions/composing-gets-me-nowhere)Students consider repeated functions that result in the original function.**Homework:**Students repeat more questions from [**Khan Academy**](https://www.khanacademy.org/math/algebra2/manipulating-functions)Exam Questions. |
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| **7** | [**UM: Function Squares**](https://undergroundmathematics.org/thinking-about-functions/function-squares/main-problem)Students consider the properties of graphs they have met | Introduce the idea of an inverse as the rearrangement of the function.[**Khan Academy: Find inverse functions**](https://www.khanacademy.org/math/algebra2/manipulating-functions#combining-and-composing-modeling-functions)Alternatively/additionally:[**TES: Inverse dominoes**](https://www.tes.com/teaching-resource/core-3-inverses-dominoes-game-6033007)**Integral Maths (old spec): Functions: Composite and Inverse functions: Activity 8:** Creating inverses: putting the stages of the rearrangement in order. | Discuss the validity of the inverse: domain of inverse = range of original.This involves exponential functionsThese functions are more complex | [**Khan Academy: Verify inverse functions:**](https://www.khanacademy.org/math/algebra2/manipulating-functions#combining-and-composing-modeling-functions)Consolidate understanding that when an inverse function is applied to the original it will return the input value.**Homework:**Exam questions finding inverses including range and domain |
| **8** **& 9** | [**TES: Functions Murder Mystery:**](https://www.tes.com/teaching-resource/functions-murder-mystery-6437381)Based on Game of Thrones students find the errors in the statements of inverse functions and domain and range | Sketching inverse graphs with domain equal to the range of the original. Establish that the graphs are reflections of each other.Alternatively:**Integral Maths (old spec): Functions: Composite and Inverse functions: Activity 7:** Establishes domain and range for inverse functions, graphs are given so line of symmetry can be established.Combining all aspects: [**Worksheet**](http://cbschoolshs.sharpschool.net/UserFiles/Servers/Server_5219792/File/Boulton/Classwork%20Inverses.pdf)Extensions/Investigations:[**UM: Can we sketch the inverse of this composite function?**](https://undergroundmathematics.org/combining-functions/r9824)Returning to Algebraic methods:[**UM: For what values of x does a function equal its inverse?**](https://undergroundmathematics.org/combining-functions/r6735) | This could be investigated using Geogebraor[**CASIO CG20 Task**](http://mei.org.uk/casio-networks)**:** use the activity under: Full A Level; Functions; Inverse Functions (Task 2) | **Plenary/Homework:**Exam question using graphed function |

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| **10** | [**Khan Academy: Modelling**](https://www.khanacademy.org/math/algebra2/manipulating-functions#combining-and-composing-modeling-functions) **with function combination:**Simple introduction to interpretation of questions from worded problems:Could be completed as a mini whiteboard task or with students coming to the board. | Students interpret the combination of functions from worded questions.[**Khan Academy: Modelling**](https://www.khanacademy.org/math/algebra2/manipulating-functions#combining-and-composing-modeling-functions) **with composite functions**Further sources of questions that could create a worksheet:[**Website questions 1**](http://www.purplemath.com/modules/fcncomp5.htm)[**Website questions 2**](http://www.analyzemath.com/CompositionFunction/applications_composition.html) | A good whole class introduction to worded composite functions: includes identification of the meaning and units of the combined function. | Plenary:Further questions from [**Khan Academy: Modelling**](https://www.khanacademy.org/math/algebra2/manipulating-functions#combining-and-composing-modeling-functions) **with composite functions** |