**Y12 AS Mathematics**

**20 Forces and Newton’s Laws of Motion 2.5 weeks**

## Author’s Note: I have assumed that as teachers you already have many standard questions available from old textbooks, and a willingness to incorporate those into your personal teaching schedule. As a Physics and Maths teacher I have tried to focus instead on a sequence through the material which gives a range of online free resources at each point and some pointers about the misconceptions which students may bring to this topic as well as what GCSE Science/Physics will have taught them. If they can crunch the numbers but have no idea how this topic relates to real life then this unit has failed to teach them anything useful.

## Teaching objectives

**a** To review prior knowledge from GCSE, including from Physics, to understand the concept of a force.

**b** To understand and use Newton’s first law. (Include Normal reaction, tension, thrust or compression, resistance.)

**c** To understand and use Newton’s second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors).

**d** To gain fluency and develop problem solving skills in the context of problems involving motion in a straight line with constant acceleration in scalar form, where the forces act either parallel or perpendicular to the motion. Resolving forces is not required. And in the context of problems involving motion in a straight line with constant acceleration in vector form, where the forces are given in **i − j** form or as column vectors.

**e** To understand and use weight and motion in a straight line under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy.

**f** To understand how to use Pythagoras’ Theorem to verify the equation of a circle.

**g** To understand and use Newton’s Third Law.

**h** To be able to deal with equilibrium of forces on a particle and motion in a straight line; equilibrium problems do not require forces to be resolved (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors)

**h** To be able to solve smooth pulley problems, either vertical or horizontal strings only.

**i** To be able to solve connected particle problems (including particles in contact eg people in lifts).

***Lots* of Useful Mechanics Links at** [**http://www.resourceaholic.com/p/mechanics.html**](http://www.resourceaholic.com/p/mechanics.html)

**Resources for advance preparation:**

MEI: Feeling Forces needs some string, ball, space to access walls to lean on. A skateboard which can tip along its length - students may have one.

Download and read through PowerPoints available at <https://haringeymath.wordpress.com/mechanics-1/> - the two on Dynamics and the one on Statics are relevant to this topic.

More resources at <http://www.mathshelper.co.uk/worksheets/> and also at <http://www.mathedup.co.uk/key-stage-5/applied-maths/mechanics-1/>

Get extra practice papers at <http://crashmaths.com/a-level-practice-papers/>

**Familiarise yourself with the common misconceptions (see session 7) and the proper answers/definitions before you start!**

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **1** | What is a force?**Use MEI** **‘Feeling forces’** [http://integralmaths.org/sow-resources.php)](http://integralmaths.org/sow-resources.php%29) gives an opportunity to confront and deal with common misconceptions about forces. | It is very important to listen to the student’s ideas at this point – get them to discuss in pairs and then write down their answers so they and you can review them later.Comparing answers round the class from the pairs is instructive; let the students challenge each other’s views so you find out who really understands what a force is. The Physics GCSE usually states ‘a force is a push or a pull’ (twisting/torsion force is not mentioned at GCSE) and that contains the essential vector idea, so encourage that when you hear it.In Physics g is always called gravitational acceleration never simply ‘gravity’ (GCSE Science says g=10 ms-2) and weight is also called gravitational force. See my notes on the MEI Feeling Forces sheet later on this document where I have added comments.Reference g being different on different planets (met in Physics GCSE, eg Moon g is 1/6 of Earth’s g because Moon is smaller mass).Check out Online Simulation: Forces and Motion the Basics <https://phet.colorado.edu/en/simulation/forces-and-motion-basics> there are different scenarios to use as plenaries or throughout for your lessons.This is a useful sheet to hand out on the differences between **weight and mass:** <http://www.thechalkface.net/resources/Weight_and_Mass.pdf>  | The inverse square law for gravitation is not required (but note this law is in Physics A2 not AS, so students will not have come across it) and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location. Extension: Applications and Modelling* A mass hangs in equilibrium on the end of a string connected to a ceiling.

What forces act on the mass?What is meant by ‘tension in the string’?What is meant by ‘breaking tension’? How can you illustrate this using a practical experiment? (safety glasses, risk assessment)You might like Interactive Simulation Hookes Law <https://phet.colorado.edu/en/simulation/hookes-law> NB A level Physics students may have studied this and Hookes Law in their first term of AS; if so perhaps they could prepare a 5 min PowerPoint presentation on this for the start of your next lesson? | Revise vectors as a plenary here: <https://www.tes.com/teaching-resource/maths-vectors-starter-plenary-powerpoint-6161352>Standard textbook questions may be used, drawing forces on different objects and finding resultant force. Ensure practice in identifying and naming (Include Normal reaction, tension, thrust or compression, air resistance, frictional resistance, gravitational force / weight.)Internet Research: Write down all three of Newton’s Laws word for word (this is a copying exercise from internet research or textbook). This will be useful for reference in later lessons.Powerpoint Use: Ask groups of three to prepare a 5 min Powerpoint on each of the following topics:* Second Law when resultant force is not zero (give to weaker students)
* Second Law when resultant force is zero = Equilibrium (weaker/middle students)
* Third Law (brighter students)
* Pulleys and Connected Particles (if needed)

Internet Research: Look up gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy. Find out g for the Equator (affected by spin of Earth), for the Himalayas (does the mass of the nearby mountains have an effect sideways?), different planets, stars, black holes, moon(s) of Jupiter…Point to make is that g varies depending on where you are, but on a cosmic scale, not going from here to London… |
| **2** | Newton’s First Law; What is a Resultant force?5 mins Starter: Show Geogebra interactive Force Diagram here: <https://www.geogebra.org/m/a9q3nrbE#material/aGM3hy9c>This illustrates vector addition and also the relation between force diagram and acceleration diagram. | Calculate resultant forces. Practise vector addition.Try Ramp: Forces and Motion as an interactive inquiry session. Found here: <https://phet.colorado.edu/en/simulation/legacy/ramp-forces-and-motion> Check out the Notes for Teachers to find worksheets which go with the Phet activities, should you wish to use them,Online Addition of Forces simulationhere: <http://www.walter-fendt.de/ph14e/resultant.htm>  | Resultant force is the same phrase as net force.Insist they draw a diagram, and *add an arrow to the side to indicate their positive direction.* Add all forces in the direction of the arrow, subtract any which point the other way. The answer is the resultant force, and *a (-) sign on the resultant means the resultant force points the opposite way to their positive direction arrow.*  | **Excel:** Challenge the students to make an Excel spreadsheet where you can enter two vectors in i-j format and their spreadcheet will add them up. Too simple for them? Can they adapt their spreadsheet so the angle betweent he vector can be input as well? (the spreadsheet would need to calculate the components first…)What about three forces? Can they use **Geogebra** to make themselves some visual force diagrams?Now, can they tell a story about their forces in real life?- are they forces for hanging a picture up, or forces on a car on a hilly road, or…? |
| **3** | Resultant force is non-zero; Newton’s Second Law, F=maStart with Forces in One Dimension interactive Online: <https://phet.colorado.edu/en/simulation/legacy/forces-1d>  | A whole one-hour lesson is here including six pages workbook:: <http://www.mrbartonmaths.com/resources/standard%20unit%20pdfs/SU%20Others/O3%20-%20Solving%20Problems%20using%20Newton%27s%20Law.pdf>**Second Law Online Simulation** here: <http://www.walter-fendt.de/ph14e/n2law.htm>  | Resolving into components Interactive Simulation Online here: <http://www.walter-fendt.de/ph14e/forceresol.htm>  | Summarise your teaching and test their learning in a final session by using PowerPoint from <https://haringeymath.wordpress.com/mechanics-1/>M1.4 Dynamics - resolving forces and F=ma |

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| **4** | Resultant force is zero; equilibrium problemsStarter: **UM: Particles on the point of sliding.**<https://undergroundmathematics.org/vector-geometry/frictional-story> | Find the third force to **UM: Make it Equal** uses i-j vector notation <https://undergroundmathematics.org/vector-geometry/make-it-equal>Constant velocity up an inclined plane is of course also equilibrium as the velocity is constant and the acceleration zero. Reinforce this with **this online simulation:** <http://www.walter-fendt.de/ph14e/inclplane.htm> Gruesome Story **Touching the Void worksheet** here: <http://www.thechalkface.net/resources/slippery_slope.pdf>  | Extension work on Friciton is in this Interactive simulation <https://phet.colorado.edu/en/simulation/friction>  | Summarise your teaching and test their learning in a final session by using PowerPoint from <https://haringeymath.wordpress.com/mechanics-1/>M1.6 Statics of a Particle |
| **5** | Newton’s Third Law and connected ParticlesStarter could be talking through this **UM example:**<https://undergroundmathematics.org/vector-geometry/r8335> | <https://undergroundmathematics.org/vector-geometry/r7982> UM: **How fast do these connected particles move?**Story-Problem: **Falling Problem worksheet**: <http://www.thechalkface.net/resources/Falling_Problem.pdf>  | Equilibrium of three forces linked to pulleys Online Simulation here: <http://www.walter-fendt.de/ph14e/equilibrium.htm> Will my **Landrover get out of the ditch? Worksheet here:** <http://www.thechalkface.net/resources/Breaking_Strain.pdf>  | Summarise your teaching and test their learning in a final session by using PowerPoint from <https://haringeymath.wordpress.com/mechanics-1/>M1.5 Dynamics for connected particles |
| **6** | Misconceptions | Check those misconceptions now by seeing whether your students can answer these three misconceptions of Newtons Laws now: Play each one and let the students explain what the real law is and why everyday life doesn’t look like this: Youtube here: <https://www.youtube.com/watch?v=Yf0BN0kq7OU> Those third law forces are equal: Youtube here: <https://www.youtube.com/watch?v=x45bXUwBgE4> The springs compress the same even though the movements are different, showing the forces are the same. One minute.Return to the initial thoughts of the students in Session 1: would they reword those answers now?Practise all kinds of questions from old textbook resources. | Stretch your students with this M1 ideas worksheet: <http://www.thechalkface.net/resources/M1_Ideas_Test.pdf> but remove the answers from the end of it first!Can they tackle the difficult **Zorb problem – worksheet here:** <http://www.thechalkface.net/resources/zorb.pdf>  | Students may like this Key Skills checklist for M1 topics but you may need to update it: <http://www.thechalkface.net/resources/M1_Checklist.pdf> Round off the topic with these **Essential Facts Cards online here:** <http://www.thechalkface.net/resources/M1_Essentials.pdf>  |
| **7** |  | Revision and Topic Test  |  |  |

**AC’s Notes on MEI resource Feeling Forces**

‘Feeling forces’ (which can be found at <http://integralmaths.org/sow-resources.php>) is an excellent introduction to simple experiments in Mechanics. It gives an opportunity to confront and deal with common misconceptions about forces.



**AC’s Notes on ‘Feeling Forces’**

1. You hope the student will say the floor is supporting me, and the idea of a normal reaction force (Newton’s third law) is suggested. However if you try to stand on a swimming pool then the normal reaction is not sufficient to support you. In the end physicists say it’s all down to the electrostatic forces between the atoms of your feet and the floor, but for now simply note that the floor supports you perpendicular to the floor. Get a skateboard and you’ll see that it supports you at right angles *to its own surface. Normal means perpendicular to the floor, whatever angle to the horizontal the floor may be.* Lifting one leg up means the force has to all go through one leg; this is more about pressure than force, you may wish to skip this one. Students will know about their centre of gravity/mass from GCSE Physics and are likely to use that in explanations. Where this is disastrously confusing as a teaching aid is in using the human *body* to feel forces…because our muscles get tired from stationary activities such as standing on one leg or holding a shopping bag for any length of time; in physics work is only done when a force actually moves through a distance. Explain this to students who mention aching joints or tired limbs.
2. Is presumably aiming at ideas around equilibrium, balanced forces. Keep this activity brief for reasons as explained. And say that the force from the floor **balances** or **is equal and opposite to** the force on their feet. Never say a force ‘moves backwards’ into their feet, forces just don’t work that way. But make the student’s body tilt, and you will have a great example of i-j vectors balancing; extend this by drawing a simple vector diagram (vertically weight from their cog and normal from their feet, horizontally into the wall and the normal from the wall) on the board and get them to copy it down, noting that sum of forces up = sum of forces down, sum of forces left = sum of forces right). The students will know from GCSE about the sum of the moments anticlockwise needing to balance the sum of moments clockwise as well. Don’t get too bogged down in that side turning.
3. This is just to get the words tension and compression into the discussion, I think. Ask for two more names of forces, you want to get engine thrust (of car, motorbike etc) and resistance (as in air resistance /drag and friction/resistance to motion) as well. Point out that every force has a direction – forces are vectors. Revise what a vector is and what a scalar is if necessary.
4. I wouldn’t touch with a bargepole any activity which requires a student to be weighed on scales. Delete that one!

**MEI’s Commentary**

This is an exciting part of the specification where students can apply the ideas developed from as early as the 17th century to many everyday scenarios. Students will learn how to combine the forces acting on a particle to obtain a single equivalent force; they will learn the condition for a system of forces applied to a particle to leave it at rest (or moving with a constant velocity); they will learn when a system of forces will cause an object to accelerate; in the latter case they will be able to determine the magnitude and direction of the acceleration

A key requirement for success is that students develop very sound understanding of the basic ideas and use efficient techniques. Many of them will have come across some of this work already but have a number of wrong ideas that have to be elicited and challenged.

It is also important that students properly understand the significance of many common modelling assumptions. For instance:

Why do we say that a string is light? This may be so that its weight need not be considered or so that it can be modelled when taut as straight even when not vertical.

Above all, whether the system of forces is in equilibrium resulting in no motion or constant velocity or there is a resultant force causing acceleration, students must always remember that the principles used are all statements about the behaviour of *vector* quantities.

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| **Forces and Newton’s laws of motion (AS)** | **Time allocation:**  |
| **Pre-requisites*** No specific knowledge is needed beyond GCSE, though GCSE Science is useful
* Simple linear and simultaneous equations
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| **Links with other topics** * Connection to Kinematics, vectors and scalars quantities
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| **Questions and prompts for mathematical thinking*** Two identical objects are connected to the ends of a light elastic string which passes over a fixed pulley as shown. What happens if the system is released from rest?
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| **Applications and Modelling*** A mass hangs in equilibrium on the end of a string connected to a ceiling.

What forces act on the mass?What is meant by ‘tension in the string’?What is meant by ‘breaking tension’? How can you illustrate this using a practical experiment? |
| **Common Errors** * Inconsistency in identifying which direction is positive when applying Newton’s Second Law
* Getting the correct direction for tension in a string
* Omitting a force e.g. normal reaction in a diagram or performing calculations without a sound diagram
* Given the weight, , using for a force or in Newton’s Second Law

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