**Y12 AS Mathematics**

**13 Exponentials and Logarithms 1.5 weeks**

## Teaching objectives

**a** Know and use the function $a^{x}$ and its graph, where *a* is positive

**b** Know and use the function $e^{x}$ and its graph

**c** Know that the gradient of $e^{kx}$ is equal to $ke^{kx}$ and hence understand why the exponential model is suitable in many applications

**d** Know and use the definition of $log\_{a}x$ as the inverse of $a^{x}$, where *a* is positive and $x\geq 0$ Know and use the function $ln x$ and its graph.

**e** Understand and use the laws of logarithms:

**f** Solve equations of the form $a^{x}=b$

**g** Use logarithmic graphs to estimate parameters in relationships of the form $y=ax^{n}$ and $y=kb^{x}$, given data for *x* and *y*

**Resources for advance preparation:**

**Indices**

**Compound Interest**

**Plotting points on a scatter graph- determine gradient and intercept of the line of best fit.**

**Graph transformations**

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **1** | **Exponential graphs**[**UM: See the Power**](https://undergroundmathematics.org/exp-and-log/see-the-power)Questions: What does the whole number part of the power often tell us about the value of yy?How is this whole number part of the power connected with standard form?How can you use the given information? | Draw the graph of y=2x and y=3xWhat will y=5x look like? What are the key similarities and differences with these graphs?Students to explore graphs of different powers including fractional and negative. Students to make key notes. Students to use DESMOS to check graphs and predictions. Introduce y=ex by getting students to look at the gradient functions of the above graphs using DESMOS. Where does the gradient function match the actual function? (Trial and improvement.)Introduce logs as an equivalent way of writing a number with a power - demonstrate its usefulness. A nice consolidation activity: [**UM: Log Lattice**](https://undergroundmathematics.org/exp-and-log/log-lattice) | Y=axWhat happens when x is big? Small? Negative? Fractional? EXT: sketch the graph of a) 10e-xb) e-1/3xc) 7e2x | The graph y=kax passes through the points (1,6) and (4,48).Find the values of the constants k and a. The function f is defined as f(x)=e0.2x (x belongs to the real numbers) Show that the tangent of the curve at the point (5, e) goes through the origin.  |
| **2** | **Logs & Exponentials**[**Standards Unit (A13)**](http://www.mrbartonmaths.com/resources/standard%20unit%20pdfs/SU%20Algebra%20Lessons/A13%20-%20Simplifying%20Logarithmic%20Expressions.pdf)True or False activity. Students to use mini whiteboards- quick assessment from last lessons learning.  | (If further consolidation is needed students to complete the jigsaw also in the standards unit). Odd One Out activity- page 8 of standards unit. Writing as a single logarithm. (resource) | STARTER EXT: Fill in the blanks from page 3 of standards Unit A13. EXT: [**UM Can we find bounds?**](https://undergroundmathematics.org/exp-and-log/r9381)EXT: [**Integral: Problem Solving Shorts 3**](https://integralmaths.org/samples/Problem_solving_shorts%20_samples.pdf)  | [**Risp 31: Building log equations**](http://www.s253053503.websitehome.co.uk/risps/risp31.html)Homework: Consolidation activity on writing as a single logarithm.  |
| **3** | **Solving log equations**Pair activity. **[UM: proving laws of logs](https://undergroundmathematics.org/exp-and-log/proving-laws-of-logs)**Students to summarise the laws of logs.  | Solving log equations.Show some key examples building in difficulty. (Building to equations which are equivalent to quadratics). Resource | Support: Practise solving quadratic equations first. EXT: Solve 3xe4x-1 =5Giving your answer in the form a+lnb/c+lnd | [**UM: Changing bases**](https://undergroundmathematics.org/exp-and-log/changing-bases)Project the graph of: y=3 + ln(4-x) (without any values).State the coordinates of the x and y intercept. Homework: routine questions from old resources |
| **4** | **Solving log equations 2**Starter: Give a list of different log equations- students have to group them into those which can be solved via the ‘quadratic’ method and those which don’t. Students then to solve 5 of the questions to challenge their learning.  | Further work on solving log equations:[**UM: Simultaneous log equations**](https://undergroundmathematics.org/exp-and-log/r5462)[**UM: Solving more difficult log equations**](https://undergroundmathematics.org/exp-and-log/r8260)[**UM: Can we solve?**](https://undergroundmathematics.org/exp-and-log/r7845) | Support and EXT available on the UM website- use the suggested questions.  |  |
| **5 & 6** | **Exponential Modelling**. Students to work in pairs to think of where we may use exponentials to model real life situations. Collate ideas as a class.  | Key Example- Density of a pesticide [**TES: exponential growth and decay**](https://www.tes.com/teaching-resource/exponential-growth-and-decay-6153459)[**MEI: Benford’s Law**](http://mei.org.uk/files/papers/c407ja_1jd8q.pdf#page=10)Get some data from your Biology colleagues (and/or geography, physics, economics…) and get students to plot log y against x to find a model to fit. | Support: Calculator use. Prompt when t=0. What happens when we differentiate ekx? | Homework: Routine questions from **new** textbooks or sample assessment materials on modelling.(Remember that you can use resources from all specifications, so no need to plunder short supplies of full practice papers from your own board.) |