**Y12 AS Mathematics**

**9/17/18 Binomial Expansion, Binomial Distribution and Hypothesis Testing 3 weeks**

## Teaching objectives

**a Understand and use the binomial expansion of** $(a+bx)^{n}$ **for positive integers *n***

**b Use the notations *n*! and *n*C*r***

**c Understand and use Binomial probabilities**

**d Understand and use other simple discrete probability distributions**

**e Know the mean and variance of the Binomial Distribution**

**f Understand apply the language of statistical hypothesis testing in the context of the Binomial Distribution (null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value)**

**g Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context**

**h Understand that a sample is being used to make an inference about the population**

**i Appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis**

**Resources for advance preparation:**

[**Using Binomial Probabilities**](http://wirksworthii.nottingham.ac.uk/Improv_Learning_Maths/print/S8.pdf)– per pair: card sort A and probability cards B

[**Integral: Binomial Distriubtion Hexagonal Jigsaw**](https://2017.integralmaths.org/pluginfile.php/1009/mod_book/chapter/94/BinomialProbHexJig.pdf)*–* one set per pair or group

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **1** | **Investigating counting**Have 5 otherwise identical books of each of 2 different colours (eg A level textbooks for statistics and pure).Take 2 ‘green’ books and 3 ‘blue’ books. How many different ways are there of arranging these books on the shelf?Get students to investigate for different combinations of 5 books, then to work backwards through 4, 3, 2 and 1 books, then to generalise. | Some students may have met Pascal’s Triangle before. It provides a wealth of useful extension work.[**Risp 32: Exploring Pascal’s Triangle**](http://www.s253053503.websitehome.co.uk/risps/risp-32.pdf) | Discussion of results, generalisations and proofs. For example, why is *n+1*C*r* = *n*C*r*-1+ *n*C*r* |
| **2** | Students multiplying out brackets:$(x+y)^{2}$, $(x+y)^{3}$, $(x+y)^{4}$ etc | Discussion of results from starter – hopefully some students at least will make a link with the previous lesson. Get students to show how this relates to the counting activity from the previous lesson.Formalise the use of the Binomial Expansion with some standard examples, including eg (1 – 2x)6, then extend to working out eg (0.99)6Use technology to investigate the accuracy of expansions as you include more powers of x, eg, [**Integral Binomial Expansion Spreadsheet**](http://mei.org.uk/integrating-technology) | Common errors include raising only part of the term to the appropriate power (ie ignoring coefficients and/or negative signs). | **Homework:**Routine practice from old resources, or **Integral Exercise level 1** |

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| **3** | Quick questions on straight forward Binomial Examples | Use a range of problem solving activities to consolidate:[**UM: Can we estimate a difference of eighth powers?**](https://undergroundmathematics.org/counting-and-binomials/r6503)**Integral: Activity 1: Problem Solving Shorts** | Extension: [**UM: Can we find an approximation to √5 + √3?**](https://undergroundmathematics.org/counting-and-binomials/r7477) | How would you explain why the coefficient of x7 in (2+x)10 is 10C7 x 23?Change one number in (1+x)4 so that the coefficient of x in the expansion is 32.Give me two examples of binomial expansions in which all the coefficients are odd.**Homework:**More complex questions from old resources or **Integral Exercise level 2** |
| **4** | Investigation: Students to draw a probability tree for throwing a 5 on a fair die 3 times. Where X is the number of 6s thrown find P(X=0), P(X=1), …..P(X=3), P(X=r).They should spot pattern. | Link with Binomial expansion and introduce terminology. Find probabilities by hand (to consolidate understanding) and on calculators.Could use: [**MEI: Binomial Experiment**](http://mei.org.uk/files/sow/16-binomial-distribution-res.pdf) (full lesson plan) or [**MEI: Falling balls**](https://phet.colorado.edu/sims/plinko-probability/plinko-probability_en.html) | Bringing in terminology: Let X be the random variable ' Number of 5s thrown in 3 throws' helps weaker students formulate more difficult problems.NB avoid counting 6s or 1s to reduce confusion between scores and probabilities! | **Homework:** Routine practice using calculators to calculate probabilities. |
| **5** | [**Using Binomial Probabilities**](http://wirksworthii.nottingham.ac.uk/Improv_Learning_Maths/print/S8.pdf) – unit S8 from the DfE Standards Unit. | This is a *brilliant* lesson plan! Students struggle with the language for cumulative probabilities, and these activities help you to address the issues | Plenary session included in lesson plan**Homework:** Routine practice using old resources. |
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| **6** | Give students a card with four properties of binomial distributions on. Put different scenarios on the board and ask them to identify whether Binomial or not. For example ‘Throwing a dart until you get a bullseye’ | Explore the mean and variance of various Binomial Distributions. (Not strictly needed for AS, but this seems to be the best place to introduce it.)Proving E(X) = np is quite straight forward for specific values of n. See [**MSV40: The Binomial Mean and Variance**](http://www.s253053503.websitehome.co.uk/msv/msv-40.html) (but see if students can develop this for themselves, with or without your assistance) | Bizarrely, the general formulae for the mean and variance of a probability distribution is not in the specification, although they *do* need to know that E(X) = np and Var(X) = npq for the Binomial Distribution when doing Normal Approximations in Y13. However, the general formula, at least for Expectation, is straight forward to derive, so why not do it anyway? | [**Integral: Binomial Distriubtion Hexagonal Jigsaw**](https://2017.integralmaths.org/pluginfile.php/1009/mod_book/chapter/94/BinomialProbHexJig.pdf) to consolidate binomial probabilities from previous lesson:**Homework:** [**Integral Exercise level 1**](https://2017.integralmaths.org/pluginfile.php/934/mod_resource/content/0/assb1ax_level1.pdf) or Routine practice using old resources. |
| **7** | Further consolidation using these resources:[**Binomial Expansion Lock Game**](https://www.tes.com/teaching-resource/binomial-expansion-lock-game-6386969) (Resourceaholic)Oh dear – all the other resources I found were very routine and not at all problem solving! Please email me if you find any good ones. |  | **Homework:** [**Integral Exercise level 2**](https://2017.integralmaths.org/pluginfile.php/935/mod_resource/content/0/assb1ax_level2.pdf) or Routine practice using old resources. |

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| **8** | Display the following:* Striders think their crisps taste better than supermarket brands.
* They’ve done research that says that 8 out of 10 people prefer Striders crisps.
* They use this in their advertising.
* Staylily supermarket think they’re wrong. They want to challenge the advertisement.
* Staylily think that fewer than 8 out of 10 people prefer Striders crisps.

Ask students how they would investigate.Be prepared with 2 makes of crisps in bowls labelled A and B. (Use the leading brand and a quality supermarket brand so that the difference is not too great!) Students to taste and put slips marked with preference in box.* Idea: work out the probability of getting this result *if the Striders claim is correct.*
* If this probability is very small, we could reasonably assume their claim is incorrect.
	+ What do we mean by ‘very small’?

Do the calculations…* + Note that for a sample size of eg 20, all of the probabilities are rather small. That means any particular result is not that likely, even exactly 16 out of 20! So we group similar results together
	+ We look at the probability of getting *at least* our result – ie, our result *or one more unlikely*.
 | Make sure students understand they are not proving anything but providing evidence.Quality discussion of concepts here will make all the difference later! Keep it informal and based on understanding.**Alternatively, you could use the starter activity to *introduce* the Binomial Distribution.** | If you can, look at data for a much larger group – over 100 if possible. (You could collect this from your y7 classes in advance!) You’ll need to do some hand-waving to calculate the probabilities using a Normal approximation (they’ll be able to do this themselves in y13!)Then ask them to complete:* The probability of getting our result or one more unlikely is…
* This is very low/not very low
* So our assumption (that 8 out of 10 people prefer Striders crisps) is unreasonable/reasonable
* We conclude…

**Homework:**Students will not yet be ready for homework on Hypothesis Testing, so give them more complex questions on Binomial Distribution from old resources or[**Integral Exercise level 2**](https://2017.integralmaths.org/pluginfile.php/935/mod_resource/content/0/assb1ax_level2.pdf) |
| **9** | [**Integral: Multiple Choice Quiz**](https://2017.integralmaths.org/mod/book/view.php?id=917&chapterid=96)You could use your own quiz instead, perhaps using a subject that *some* of your students know about. | Formalise the procedure for conducting a hypothesis test, introducing all the key vocabulary. Use one of the Large Data Sets to make and test a hypothesis. (Remember you can use any LDS, not just the one for the specification you are doing.) | Take time to ensure students really understand what they are doing and why. | **Homework:**One or two questions done carefully will be sufficient to check students’ understanding and this stage. Use routine questions from old resources or section 1 [**Integral Exercise level 1**](https://2017.integralmaths.org/pluginfile.php/940/mod_resource/content/0/assh1ax_level1.pdf) |
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| **10** |  | Introduce the idea of Critical Regions, again using hypotheses involving real data.[**MEI: Critical regions**](https://www.geogebra.org/m/FgSQ5WRe) technology allows students to visualise the critical region of a Hypothesis Test. | Weaker students may need more support identifying one-tail and two-tail tests. | [**Integral: Matching Critical Regions**](https://2017.integralmaths.org/mod/book/view.php?id=918&chapterid=98)**Homework:**Routine questions from old resources, or section 2 [**Integral Exercise level 1**](https://2017.integralmaths.org/pluginfile.php/944/mod_resource/content/0/assh2ax_level1.pdf) |
| **11 - 14** | Use the LDS to create your own hypotheses.Alternatively, ask students studying Biology, Psychology or Geography to come up with ideas…And test them… | This activity is challenging but interesting:[**Testing Strategy**](http://nrich.maths.org/6956) (nRich) | Routine questions from old resources, or section 2 [**Integral Exercise level 2**](https://2017.integralmaths.org/pluginfile.php/945/mod_resource/content/0/assh2ax_level2.pdf) |