**Y12 AS Mathematics**

**8 Graphs and Transformations 1 week + 4 stand-alone lessons**

## Teaching objectives

aTo review graph work from Unit 4 Equations and inequalities

b To understand and use proportional relationships and their graphs

c To understand the effect of simple transformations on the graph of $y=f\left(x\right)$ including sketching associated graphs: $y=af\left(x\right), y=f\left(x\right)+a, y=f\left(x+a\right), y=f(ax)$

d To know and use graphs of exponential and logarithmic functions

e To understand transformations of trig graphs

**Resources for advance preparation:**

For many of the activities (particularly UM and Don Steward) it may help to screen clip the questions onto a power point to improve visibility when projected.

[**UM: Picture the process I**](https://undergroundmathematics.org/thinking-about-functions/picture-the-process-i)

 “Process, graph and equation cards to cut out” need to be printed and cut out. (Or just print and label but don’t cut out.)

[**UM: Curve match**](https://undergroundmathematics.org/thinking-about-functions/curve-match)**;** [**NRICH: Parabolic patterns**](http://nrich.maths.org/773) **;** [**NRICH: Parabolas again**](http://nrich.maths.org/791)**;** [**NRICH: Cubics**](http://nrich.maths.org/802) **;** [**UM: Transformers problem**](https://undergroundmathematics.org/combining-functions/transformers/problem)**;** [**DonSteward: Daylight/Weather Sines**](http://donsteward.blogspot.co.uk/search/label/trig%20function)Colour copies of the graph images need to be printed for each pair/group.

[**UM: Name that graph again**](https://undergroundmathematics.org/combining-functions/name-that-graph-again)Student starting points need to be expanded and either printed for pairs/groups or clipped and added to a power point.

[**DonSteward: Cubics**](http://donsteward.blogspot.co.uk/2012/07/cubics.html)Print the third slide for each pair/group

[**MEI-AS-SoW-Understanding Transformations**](http://mei.org.uk/files/sow/07-graphs-and-transformations-res.pdf) **;** [**UM: Sketching a cubic (MAT question)**](https://undergroundmathematics.org/polynomials/r6612)**; [AQA: Transforming functions](https://allaboutmaths.aqa.org.uk/1028);** [**UM: Can we identify the right sketch of a transformed function?**](https://undergroundmathematics.org/combining-functions/r7119) ; [**UM: How do these transformations change the graph of f(x)?**](https://undergroundmathematics.org/combining-functions/r9673) **;** [**DonSteward: Graph Transforms**](http://donsteward.blogspot.co.uk/search/label/graph%20transforms)

Print worksheets per pair or email to students as HW

[**UM: Approaching asymptotes**](https://undergroundmathematics.org/thinking-about-functions/approaching-asymptotes)Need to print descriptions and graphs (which can be cut up as cards if you wish)

[**UM: Worth 1000 words Problem**](https://undergroundmathematics.org/thinking-about-functions/worth-1000-words/problem)Need to print sketch graphs and function cards

[**NRICH: Tangled Trig Graphs**](http://nrich.maths.org/6481): Colour copies of the graphs are needed per pair/group

[**Standards Unit: Exploring trigonometrical graphs**](http://www.mrbartonmaths.com/resources/standard%20unit%20pdfs/SU%20Algebra%20Lessons/A12%20-%20Exploring%20Trigonometrical%20Graphs.pdf) Sheet 1 enlarged to A3 and Card set A for each group; Sheet 2 for each student or to be projected.

**The first four lessons of this unit are to be taught as a block. The next 4 are to be spaced at your discretion to allow for review of this topic and to emphasise the relationship to other units when they have been covered (such as exponentials).**

**Geogebra/ Desmos/ graphical calculators should be used throughout all the activities as either a way to demonstrate, or to help explore. You may need to allow time to explicitly teach use of software if not yet used.**

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **1** | [**UM: Picture the process I**](https://undergroundmathematics.org/thinking-about-functions/picture-the-process-i)Sketch graphs to represent real processesOR [**NRICH: Parabolic patterns**](http://nrich.maths.org/773)[**NRICH: Parabolas again**](http://nrich.maths.org/791) | [**UM: Curve match**](https://undergroundmathematics.org/thinking-about-functions/curve-match) Consider graphs of five functions and their behaviour for 0<x<1You will need colour copies of the graphs image.Students will be familiar with function notation, reciprocal graphs and proportion from Higher GCSE though they will require further experiences of these topics and a more rigorous treatment.Asymptotes can be explored numerically alongside the graph - the idea of approaching a limiting value is met in many topics later in the course. |  | [**DrFrostMaths: Ch4(Slides 3-25)**](http://www.drfrostmaths.com/resources/resource.php?rid=272)Free powerpoints with examples and modelling. References Edexcel textbook. Students can be registered free to practise Edexcel past paper questions or relevant MAT questions. |
| **2** | [**UM: Sketching a cubic (MAT question)**](https://undergroundmathematics.org/polynomials/r6612)Exploring features of cubic graphs and relating to factor theoremOR [**NRICH: Cubics**](http://nrich.maths.org/802) | [**UM: Can you find... cubic edition**](https://undergroundmathematics.org/polynomials/can-you-find-cubic-edition)Find cubic curves that meet given graphical criteria. Reviews factor theorem and emphasises key features of polynomials and how they relate to graphical features.**Encourage students to consider the shape and features of the graphs and try to distance them from the need to plot.**  |  | [**DonSteward: Cubics**](http://donsteward.blogspot.co.uk/2012/07/cubics.html)Explore graphs of cubic functions |

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| **3** | [**UM: Name that graph again**](https://undergroundmathematics.org/combining-functions/name-that-graph-again)Looking at ways to identify a quadratic equation from a graph | Use the dynamic facility of graphing software to develop a feel for the effect of transformations; it can be used to cover many examples in a short time.Reflections and translations of graphs will have been met at Higher GCSE.[**UM: It's a matter of perspective**](https://undergroundmathematics.org/combining-functions/its-a-matter-of-perspective) Consider features of a graph as it is transformed[**MEI-AS-SoW-Understanding Transformations**](http://mei.org.uk/files/sow/07-graphs-and-transformations-res.pdf)Uses piecewise graphs and tables of values to reinforce how functions are transformed.[**Autograph Activities**](http://www.autograph-maths.com/challenges/tag/transformation-of-functions/)You would need Autograph licenses to use these activities | Students should recall translations and reflections of functions from GCSE (but perhaps not if not in highest attaining groups- begin by checking perhaps using **Diagnostic Questions Higher GCSE Transforming functions**). Use GCSE resources to support those who need to catch up.Integral (Graphs and Transformations) has exercises and Geogebra animations to use for exploration as well as Section tests for assessment. | [**AQA: Transforming functions**](https://allaboutmaths.aqa.org.uk/1028)(From All About Maths which requires registration but is free.)GCSE questions on transforming functions[**UM: Can we identify the right sketch of a transformed function?**](https://undergroundmathematics.org/combining-functions/r7119)(MAT question)[**UM: How do these transformations change the graph of f(x)?**](https://undergroundmathematics.org/combining-functions/r9673) (A level question 1988)[**DrFrostMaths: Ch4(Slides 26-34)**](http://www.drfrostmaths.com/resources/resource.php?rid=272)Free power point with examples and modelling. References Edexcel textbook. Students can be registered free to practise Edexcel past paper questions or relevant MAT questions.[**CIMT: Graph transforms**](http://www.cimt.org.uk/projects/mepres/alevel/pure_ch4.pdf)Online text book with activities, examples and exercises.[**Teachitmaths: Graph transformation blitz**](https://www.teachitmaths.co.uk/resources/ks5/transformations-of-functions/graph-transformation-blitz/17238)Basic transformation of functions worksheet **Bring on the Maths (can be purchased from Kangaroo Maths) Investigating Graphs I and II** |
| **4** | [**UM: Transformers Warm-up**](https://undergroundmathematics.org/combining-functions/transformers)Consider pairs of graphs and categorise them by thinking about transformations | [**UM: Transformers problem**](https://undergroundmathematics.org/combining-functions/transformers/problem)Using the pairs of graphs from the starter students try to write one function in terms of the other by considering the transformation involved.You will need to have colour copies of the graphs and to project the question prompts to guide their thinking.[**DonSteward: Graph Transforms**](http://donsteward.blogspot.co.uk/search/label/graph%20transforms)[**TES: Venn Diagram of Transformation of Functions**](https://www.tes.com/teaching-resource/venn-diagrams-16-transformation-of-functions-11074231) |
|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **5** | [**UM: Translating or not?**](https://undergroundmathematics.org/combining-functions/translating-or-not)Exploring graphs of rational functions  | [**UM: Approaching asymptotes**](https://undergroundmathematics.org/thinking-about-functions/approaching-asymptotes)Discuss definition of an asymptote and consider possible definitions by looking at 16 different curves. | **New topic in A level maths so may need to look at old Further Maths resources for practise exercises.** |
| **6** | **Re-cap a variety of graph shapes**[**UM: Worth 1000 words Warm-up**](https://undergroundmathematics.org/thinking-about-functions/worth-1000-words) Consider 4 different functions and what to consider if trying to sketch them. | [**UM: Worth 1000 words Problem**](https://undergroundmathematics.org/thinking-about-functions/worth-1000-words/problem)Consider possible graphs of a rational function. Sketching such graphs is beyond AS maths but the point is to use an unfamiliar function to emphasise the difference between plotting and sketching. Discuss which features of a graph might be important to include and which might not. | **NO GRAPHING SOFTWARE FOR THIS ONE (until maybe right at the end)** | Integral or old textbook resources can be used to consolidate these topics. |
| **7** | **For after Unit 13 on Exponentials and Logarithms**[**UM: When does this exponential function equal this linear one?**](https://undergroundmathematics.org/exp-and-log/r8013) | [**NRICH: Cobalt Decay**](http://nrich.maths.org/6560) **OR** [**NRICH: Drug Stabiliser**](http://nrich.maths.org/6457)Consider real situations where exponential decay is modelled.Kangaroo Maths Probing questions* Draw the graph of y = 1x. Convince me it is an exponential function
* show me the graph of an exponential function. And another, and another, …
* Always/Sometimes/Never: The graph of an exponential function, y = kx for positive values of k, does not intersect with the x-axis
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| **8** | **Re-cap Trig graphs and link to transformations of functions and modelling**[**NRICH: Tangled Trig Graphs**](http://nrich.maths.org/6481) | Sketch transformations of the trigonometric graphs o understand changes to period o understand changes to maximum and minimum values[**Standards Unit: Exploring trigonometrical graphs**](http://www.mrbartonmaths.com/resources/standard%20unit%20pdfs/SU%20Algebra%20Lessons/A12%20-%20Exploring%20Trigonometrical%20Graphs.pdf) Standards unit lesson on exploring trig graphs(assuming not used in trig unit)[**DonSteward: Daylight/Weather Sines**](http://donsteward.blogspot.co.uk/search/label/trig%20function)Use real weather data to find a sine function that fits the graphs. |  |