**Y12 AS Mathematics**

**7 Polynomials 1 week**

## Teaching objectives

**a Review knowledge of simplifying from GCSE**

**b Learn efficient ways to multiply more complex brackets**

**c Develop intuition about the link between factored form and appearance of functions**

**d To understand how to divide a polynomial by a linear factor, and why/how the procedure works**

**e To review GCSE substitution, bringing in new technology**

**f To understand the link between substituting into an expression and finding zero, and factorising the polynomial.**

**g To gain fluency and problem solving skills in the context of polynomial factorisation**

**h To be able to use the fancy calculators to quickly gain intuition about questions and check solutions**

**Resources for advance preparation:**

[**Underground Maths Function Builder I**](https://undergroundmathematics.org/combining-functions/function-builder-i/download/problem.pdf) and [**the cards to go with it**](https://undergroundmathematics.org/combining-functions/function-builder-i/components/cards.pdf)– Two sheets between 2

[**Underground Maths Divide it up Warm up**](https://undergroundmathematics.org/polynomials/divide-it-up/download/warm-up.pdf)

[**Polynomial division using grids worksheet**](https://www.tes.com/teaching-resource/dividing-polynomials-not-by-long-division-6343581) – Pages 2-3 are the most useful, but 1 is good supporting material, and 4-5 would make a nice homework

[**Factor Theorem True/False questions**](https://drive.google.com/file/d/0B9L2lYGRiK2bMmFYQ0YzTDlBQnc/view) – Pages 4 and 5 of this, but there are a number of other nice activities in the same file.

[**Factor and Remainder Theorem Exam Questions**](http://www.mathsgenie.co.uk/resources/c2-factors-and-remainders.pdf) - Remainder Theorem is not required, so the questions to answer would need to be specified.

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **1** | [**Underground Maths Function Builder I**](https://undergroundmathematics.org/combining-functions/function-builder-i/download/problem.pdf)Students can demonstrate that their algebraic manipulation skills are still good by writing their answers on a projected blank task outline. Once full, look at the patterns in the rows and columns and discuss why they are there (e.g. line + line = line).Key idea: the *degrees* of the two polynomials and how they combine under different operations. | If pupils remember about sum/product in Quadratics and how the roots and factored form are linked (if not, terrify them with [**this**](https://www.tes.com/teaching-resource/quadratic-skills-problem-solving-task-11689600) for homework), let’s move on to higher degree polynomials.Have a play with this [**Geogebra App**](https://www.geogebra.org/m/MnsaAW8p). Can you find a cubic with 3 roots? 2 roots? 1 root? No roots? Why/Why not/How? [**Quartics here.**](https://www.geogebra.org/m/BJtGKWWE)How can we find the factorisation of a cubic (or anything higher)? Does having it partially factorised help? e.g. $x^{3}-7x-6=(x^{2}+3x+2)(x-3)$ or $x^{4}-2x^{3}-13x^{2}+38x-24=(x^{2}-4x+3)(x^{2}+2x-8)$. What is the link between the roots, the graph, and the factorisation?If we can break it down into quadratic or linear factors, it should be possible. [**[Quadratic factorisation puzzles for a reminder/extension]**](https://www.tes.com/teaching-resource/quadratic-areas-puzzle-11689598).  | Geogebra/Desmos/graphical calculators could be used throughout all the activities as either a way to demonstrate, or to help.The idea we are trying to get at is that the factors match with the roots, and the roots match the places where the graph is zero. Hence, if we can find the zero points, we can factorise. | By substitution, find the roots of these equations (all are integers between -10 and 10).$$y=x^{3}-6x+11x-6$$$$y=x^{3}-2x^{2}-5x+6$$$$y=x^{3}-5x^{2}-9x+45$$$$y=x^{4}-17x^{3}+101x^{2}-247x+210$$$$y=x^{4}+19x^{3}+92x^{2}-28x-480$$There is a good discussion that could be had here (start of the next lesson?) on the shortcuts that people took for this, either with technology or mathematical insight. |
|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| **2** | [**Underground Maths Divide it up Warm Up**](https://undergroundmathematics.org/polynomials/divide-it-up/download/warm-up.pdf)Links grid method for multiplying with factorising numbers, then puts in a little bit of algebra, then a lot of algebra. | MWB activityDiscussion: Thinking about yesterday, why should cubic/linear=quadratic?What about $\frac{6x^{3}+19x^{2}+16x+4}{3x+2}$?$\frac{4x^{4}+3x^{3}+2x+1}{x^{2}+x+2}$? What does the grid look like here?[**Structured worksheets on this.**](https://www.tes.com/teaching-resource/dividing-polynomials-not-by-long-division-6343581) The first page covers the multiplying again, the next two do more dividing, with reducing scaffolding, and the last two are more like exam questions. | When there are negative coefficients involved, things will undoubtedly go wrong. A progression using grids and questions such as $(20-1)×(10+3)$, $(30-2)×(20-3)$, $(x-2)×(20-7)$, $(x-4)×(x-2)$ and then moving to the multiplied expression and a half completed grid might help.A possible extension is to look at the polynomial division algorithm, but as the remainder theorem has been dropped, this method is probably more accessible. | How does the division link to factorisation?Calculate $\frac{x^{3}-2x^{2}-13x-10}{x+1}$ then factorise $x^{3}-2x^{2}-13x-10$.Calculate $\frac{6x^{3}+25x^{2}+23x+6}{3x+2}$ then factorise $6x^{3}+25x^{2}+23x+6$.Calculate $\frac{6x^{4}+11x^{3}-52x^{2}+4x+16}{2x^{2}+9x+4}$ then factorise $6x^{4}+11x^{3}-52x^{2}+4x+16$.Possible homework: Completing the sheets to the left – there are 5 pages of them. |
| **3** | How do we know what to divide by?What did we learn in the first two lessons?How can we combine them to factorise $x^{3}-4x^{2}-11x+30$? | Lead pupils through the example [answer is $(x-2)(x+3)(x-5)$], and then generalise the process.Extend to higher degree equations and those with an insoluble quadratic factor. How many roots *should* a polynomial of degree *n*  have? Is the factorisation always going to be unique? (Compare to Prime factorisation.)If you feel that they need to have a practice of doing this, there are some exam questions [**here**](http://www.mathsgenie.co.uk/resources/c2-factors-and-remainders.pdf), but they also include the Remainder Theorem, which is not required. | Is it necessary to consider every integer in order to find a root, or can we be more efficient in our choices?Extensions:Now you have factorised the expressions, can you draw what they look like?Problem Solving:Find all the real solutions of$$\left(x^{2}-7x+11\right)^{x^{2}-11x+30}=1$$$$\left(x^{2}-5x+5\right)^{x^{2}-11x+30}=1$$$$\left(x^{2}-7x+11\right)^{x^{2}-13x+42}=1$$There are more than you expect… | Factor Theorem True/False activity [**[Pages 4 and 5 of this]**](https://drive.google.com/file/d/0B9L2lYGRiK2bMmFYQ0YzTDlBQnc/view).How can we check answers using a fancy calculator?On the Classwiz, it is Mode A (Equations/Functions), then 2 (Polynomial), Select the degree, then type in the coefficients, pressing = between each. After that, pressing = will cycle between the roots (including complex ones), and for Quadratics also give the turning point. |