Y13 Pure Maths

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| **Proof**Understand and be able to use proof by contradiction.  |
| **Algebra**Be able to express algebraic fractions as partial fractions. Be able to simplify rational expressions.  |
| **Functions**Understand the definition of a function, and be able to use the associated language. Understand and use composite functions. Understand and be able to use inverse functions and their graphs. Know the conditions necessary for the inverse of a function to exist and how to find it. Understand and be able to use the modulus function. Be able to solve simple inequalities containing a modulus sign. Be able to use functions in modelling.  |
| **Graphs**Understand the effect of combined transformations on a graph and be able to form the equation of the new graph and to sketch it. Be able to recognise the transformations that have been applied to a graph from the graph or its equation. Be able to use stationary points of inflection when curve sketching. |
| **Parametric Equations**Understand the meaning of the terms parameter and parametric equations. Be able to convert between cartesian and parametric forms of equations. Understand and use the equation of a circle written in parametric form. Be able to find the gradient of a curve defined in terms of a parameter by differentiation. Be able to use parametric equations in modelling.  |
| **Sequences and Series**Use the binomial expansion of $(1+x)^{n}$ where *n* is any rational number. Be able to write $(a+bx)^{n}$ in the form $a^{n}(1+\frac{b}{a}x)^{n}$ hence expand $(a+bx)^{n}$ stating the validity conditions for the expansion.Be able to use binomial expansions with *n* rational to find polynomials which approximate $(a+bx)^{n}$. Know what a sequence of numbers is and the meaning of finite and infinite with reference to sequences. Be able to generate a sequence using a formula for the kth term, or a recurrence relation of the form ak+1=f(ak) Know that a series is the sum of consecutive terms of a sequence. Understand and use sigma notation. Be able to recognise increasing, decreasing and periodic sequences. Know the difference between convergent and divergent sequences. Understand and use arithmetic sequences and series. Be able to use the standard formulae associated with arithmetic sequences and series. Understand and use geometric sequences and series. Be able to use the standard formulae associated with geometric sequences and series. Know the condition for a geometric series to be convergent and be able to find its sum to infinity. Be able to use sequences and series in modelling. |
| **Trigonometry**Know and be able to use exact values of sin x, cos x, tan *x* for $x=0,\frac{π}{6},\frac{π}{4},\frac{π}{3},π$ and multiples thereof and sin x, cos x for $x=\frac{π}{4}$ and multiples thereof. Understand and use the definitions of the functions arcsin, arccos and arctan, their relationship to sin, cos and tan, their graphs and their ranges and domains. Understand and use the definition of a radian and be able to convert between radians and degrees. Know and be able to find the arc length and area of a sector of a circle, when the angle is given in radians. Understand and use the standard small angle approximations of sine, cosine and tangent. Understand and use the definitions of the sec, cosec and cot functions. Understand relationships between the graphs of the sin, cos, tan, cosec, sec and cot functions. Understand and use the relationships $tan^{2}x+1=sec^{2}x$and $cot^{2}x+1=cosec^{2}x$Understand and use the identities for $\sin((θ\pm φ))$$\cos((θ\pm φ)) $ $\tan((θ\pm φ))$Know and use identities for sin2x, cos2x, tan2x*.* Understand and use expressions for $a\cos(x)\pm b\sin(x)$ in the equivalent forms $R\sin((x\pm α))$ and $R\cos((x\pm α))$Use trigonometric identities, relationships and definitions in solving equations. Construct proofs involving trigonometric functions and identities. Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.  |
| **Calculus: Differentiation**Be able to differentiate $e^{kx}$, $a^{kx} $and ln x. Be able to differentiate the trigonometrical functions: sin kx; cos kx; tan kx for x in radians. Be able to differentiate the product of two functions. Be able to differentiate the quotient of two functions. Be able to differentiate composite functions using the chain rule. Be able to find rates of change using the chain rule, including connected rates of change and differentiation of inverse functions. Be able to differentiate a function or relation defined implicitly. Understand that a section of curve which has increasing gradient (and so positive second derivative) is concave upwards. Understand that a section of curve which has decreasing gradient (and so negative second derivative) is concave downwards. Understand that a point of inflection on a curve is where the curve changes from concave upwards to concave downwards (or vice versa) and hence that the second derivative at a point of inflection is zero. Be able to use differentiation to find stationary and non-stationary points of inflection.  |
| **Calculus: Integration**Be able to integrate $e^{kx}$, $\frac{1}{x} $, sin *kx,* cos *kx* and related sums, differences and constant multiples. Understand integration as the limit of a sum. Be able to use integration to find the area between two curves. Be able to use integration by substitution in cases where the process is the reverse of the chain rule (including finding a suitable substitution). Be able to use integration by substitution in other cases. Be able to use the method of integration by parts in simple cases. Be able to integrate using partial fractions that are linear in the denominator. |
| **Differential Equations**Be able to formulate first order differential equations using information about rates of change. Be able to find general or particular solutions of first order differential equations analytically by separating variables. Be able to interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution.  |
| **Numerical Methods**Be able to locate the roots of f(x)=0 by considering changes of sign of f(x) in an interval of *x* in which f(x) is sufficiently well-behaved. Be aware of circumstances under which change of sign methods may fail. Be able to carry out a fixed point iteration after rearranging an equation into the form x=g(x) and be able to draw associated staircase and cobweb diagrams. Be able to use the Newton-Raphson method to find a root of an equation and represent the process on a graph.Understand that not all iterations converge to a particular root of an equation. Be able to find an approximate value of a definite integral using the trapezium rule, and decide whether it is an over- or an under-estimate. Use the sum of a series of rectangles to find an upper and/or lower bound on the area under a curve. Use numerical methods to solve problems in context. |
| **Vectors**Understand the language of vectors in three dimensions.  |

Y13 Mechanics

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| **Modelling**Understand and use derived quantities and units: moments.  |
| **Kinematics**Understand the language of kinematics appropriate to motion in 2 dimensions. Know the difference between, displacement, distance from and distance travelled; velocity and speed, and between acceleration and magnitude of acceleration. Be able to extend the scope of techniques from motion in 1 dimension to that in 2 dimensions by using vectors. Be able to find the cartesian equation of the path of a particle when the components of its position vector are given in terms of time. Be able to use vectors to solve problems in kinematics.  |
| **Projectiles**Be able to model motion under gravity in a vertical plane using vectors. Be able to formulate the equations of motion of a projectile using vectors. Know how to find the position and velocity at any time of a projectile and find range and maximum height. Be able to find the initial velocity of a projectile given sufficient information. Be able to eliminate time from the component equations that give the horizontal and vertical displacement in terms of time to obtain the equation of the trajectory. Be able to solve simple problems involving projectiles.  |
| **Forces**Be able to resolve a force into components and be able to select suitable directions for resolution. Be able to find the resultant of several concurrent forces by resolving and adding components. Know that a particle is in equilibrium if and only if the resultant of the forces acting on it is zero. Know that a body is in equilibrium under a set of concurrent forces if and only if their resultant is zero. Know that vectors representing a set of forces in equilibrium sum to zero. Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium. Be able to formulate and solve equations for a particle in equilibrium: by resolving forces in suitable directions; by drawing and using a polygon of forces. Understand that the overall contact force between surfaces may be expressed in terms of a frictional force and a normal contact force and be able to draw an appropriate force diagram. Understand that the normal contact force cannot be negative. Understand that the frictional force may be modelled by 𝐹≤𝜇𝑅 and that friction acts in the direction to oppose sliding. Model friction using 𝐹=𝜇𝑅 when sliding occurs. Be able to apply Newton’s Laws to problems involving friction. |
| **Newton’s Laws of Motion**Be able to formulate the equation of motion for a particle moving in a straight line or in a plane.  |

Y13 Statistics

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| **Probability**Understand and use mutually exclusive events and independent events and associated notation and definitions. Be able to use Venn diagrams to assist in the calculations of probabilities. Know how to calculate probabilities for two events which are not mutually exclusive. Be able to calculate conditional probabilities by formula, from tree diagrams, two-way tables, Venn diagrams or sample space diagrams. Know that P(B|A ) = P(*B*) ⇔ *B* and *A* are independent.  |
| **Normal Distribution**Be able to use the Normal distribution as a model. Know the shape of the Normal curve and understand that histograms from increasingly large samples from a Normal distribution tend to the Normal curve. Know that linear transformation of a Normal variable gives another Normal variable and know how the mean and standard deviation are affected. Be able to standardise a Normal variable. Know that the line of symmetry of the Normal curve is located at the mean and the points of inflection are located one standard deviation away from the mean. Be able to calculate and use probabilities from a Normal distribution. Be able to model with probability and probability distributions, including recognising when the binomial or Normal model may not be appropriate.  |
| **Hypothesis Testing**Know that random samples of size n from $N\~(μ,σ^{2})$ have the sample mean Normally distributed with mean μ and variance $\frac{σ^{2}}{n}$. Be able to carry out a hypothesis test for a single mean using the Normal distribution and be able to interpret the results in context. Be able to identify the critical and acceptance regions.Understand correlation as a measure of how close data points lie to a straight line. Understand that a rank correlation coefficient measures the correlation between the data ranks rather than actual data values. Be able to use a given correlation coefficient for a sample to make an inference about correlation or association in the population for given p-value or critical value. |