**Y13 A Level Mathematics**

**37 Probability Distributions (Normal) 2.5 weeks**

## Teaching objectives

**a** Know the shape of the Normal curve and understand that histograms from increasingly large samples from a Normal distribution tend to the Normal curve

**b** Know that the line of symmetry of the Normal curve is located at the mean and the points of inflection are located one standard deviation away from the mean

**c** Be able to calculate and use probabilities from a Normal distribution. Know the rough distribution of the Normal e.g. 66% +/- 1sd and 95% +/- 2sd

**d** Know that linear transformation of a Normal variable gives another Normal variable and know how the mean and standard deviation are affected. Be able to standardise a Normal variable

**e** Understand, specify and use the Normal Distribution as a model

**f** Recognising when the binomial or normal model may not be appropriate

**g** Select an appropriate probability distribution for a context (with appropriate reasoning)

**h** Identify circumstances when a normal approximation to the binomial is appropriate and estimate the mean and variance

**i** Use continuity corrections to calculateBinomial probabilities when a normal estimation is used

**Resources for advance preparation:**

Answers to CIMT worksheets are available by emailing from a school email address. Please send a request, from an official educational institution e-mail address to: [feedback@cimt.org.uk](mailto:feedback@cimt.org.uk). Please include your name and the name, address and type of your institution.

**Further resources are in a separate folder**

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|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **0** |  | Pre-Module Preparation  Set students activity 1 from Chapter 8, CIMT <http://www.cimt.org.uk/projects/mepres/alevel/stats_ch8.pdf>. | This involves collecting 100 items of data from a choice of 6 activities which will give normal distributions. |  |

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| **1** | Using a histogram to find probabilities  “Normal” Powerpoint slide 1 | The more categories we break a histogram into and the more data we have, the more accurately it reflects the whole population.  <https://www.geogebra.org/m/TDgjYZb7>  The smaller the width of the groups, the closer it approximates a curve. At the limit, with an infinite amount of data, it becomes a curve. We call this the distribution.  What corresponds to finding the area of bars? How do we find the area under a curve?  <https://www.geogebra.org/m/qe6GVHeN>  We can’t integrate this function analytically, but your calculator has a table of values stored in it.  Plot histograms of the data you collected for homework on your computer. Experiment with different numbers of groups. Find the mean, median, mode, variance and standard deviation of your data. What is the probability of being within 1 standard deviation of the mean? The probability of being within 2 standard deviations of the mean? | Key idea: Understanding that probability is area for graphs of continuous normal distributions.  Key Characteristics of the Normal distribution: Symmetrical, bell shaped, mean=mode=median.  Relating this to finding probability from histograms (useful revision!)  You can use Excel, Google Sheets, Geogebra and bespoke stats packages such as minitab, SPSS to analyse data. There are plenty of online tutorials, but do plan this part in advance! | Opportunities to use the large data set while work on this.  Homework  Explore which variables are normally distributed? |

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| **2** | Report back on investigation of big data set: which variables are normally distributed? | Analytically analysing the normal distribution N(0,1) curve  Plot the graph y=(√2π)^-1 e^(-0.5x2) at intervals of 0.2 from -2.6 to 2.6. Save yourself some work by sketching it roughly first.  By differentiating the equation, find all turning points and points of inflexion. Mark these on your graph.  By using the trapezium rule between 0 and 2.5 with 10 strips of equal width, find an approximation of the integral between =/- infinity. Why is this a useful characteristic of the curve?  By using the trapezium rule between 0 and 1 with 10 strips of equal width, find an approximation of the integral between -1 and + 1. What proportion of the Normal distribution lies within 1 standard deviation of the mean?  By using the trapezium rule between 1 and 2 with 10 strips of equal width, and the answer above, find an approximation of the integral between -2 and +2. What proportion of the Normal distribution lies within 2 standard deviations of the mean? | Remember that the Normal distribution is a continuous distribution  Probabilities such as P(X = 0.4) will be zero if X is a Normal distribution, as this would mean finding the area of a line.  However, make sure you read the question carefully as there will be situations where this probability will not be zero when an approximation is being used.  Give answers to probabilities to 3 significant figures | Key questions:  How can you use the trapezium rule to find any probability for the Normal distribution?  What is the probability that x is equal to a value?  Investigation homework:  Make a list of 20 measurements which are normally distributed from as wide a range of applications as you can. E.g. Biology, Physics, Economics, Anthropology, Sports Science,… |

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| **3** | Share homework research: What types of variables are normally distributed? What types are NOT? (good start for thinking about which model to choose). | Finding probabilities for the normal distribution. Rather than using the trapezium rule, the values are stored in your calculator and in tables.  A Task exploring how to use the normal distribution function on the calculator is available on page 4 of this document:  <http://mei.org.uk/files/ict/stats-casio.pdf>  For ClassWhiz, instructions can be found in the manual, pages 31-32:  <http://support.casio.com/storage/en/manual/pdf/EN/004/fx-570_991EX_EN.pdf>  Use the calculator to complete Exercise 8B from the “CIMT stats\_ch8 Normal” worksheet <http://www.cimt.org.uk/projects/mepres/alevel/stats_ch8.pdf>  Verify that the answers are the same whether using the stats tables in the formula book or the calculator. | Use symmetry where you can  Initially make sure you are very confident in being able to manipulate standardised test scores by using symmetry.  Make good use of diagrams to illustrate your answers Draw a sketch showing the distribution and shading the area you are considering.  Discuss the minor differences between calculator and stats tables caused by rounding. Why is it appropriate to give answers to 3sf? | Consolidate using your calculator and finding <, > and <x< probabilities. |
| **4** | NRICH Task: Into the Normal Distribution  <https://nrich.maths.org/6314>  Consolidates understanding of probability as area and tackles some misconceptions. | Matching activity to find probabilities from known areas under the normal distribution:  <http://mei.org.uk/files/sow/36-probability-distributions-res.pdf>  Similar activity with graphs drawn:  **36 - Normal Curves Matching activity** | Phi(z) means p(X<=z) for Z~N(0,1) |  |
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| **5** | Start with a set of 10 items of data. Subtract 5 from them all. What happens to the mean?  Divide by 7. What happens to the variance? What happens to the standard deviation? | Demonstration of transformation  <https://www.geogebra.org/m/HIp6nzDq>  Take the data from your first lesson, subtract the mean and divide by the standard deviation for all the data. Check that the mean is now 0, and the probability of being 1 or 2 standard deviations from the mean has stayed the same.  Making Statistics Vital  <http://www.s253053503.websitehome.co.uk/msv/msv-14/msv-14.pdf>  Number puzzle which involves specifying Normal distributions, probabilities and transformations. | Define your variables Carefully define your non-standardised variable with X or Y or….. (but of course not Z).  Remember to specify N(mean, variance) but use sd. Can specify N(mean, sd2) to help.  Be careful to distinguish between values of Z and values of X Confusion with notation makes it harder for you to be awarded method marks. Show clearly how you are standardising values.  Write down clear probability statements Again you are more likely to receive method marks if your statements are easy to read. | **MSV-26 Adding 2 Normal Distributions (beyond syllabus but interesting extension)**  [**http://www.s253053503.websitehome.co.uk/msv/msv-26.html**](http://www.s253053503.websitehome.co.uk/msv/msv-26.html)  **36 - Normal Distribution Questions**  Section 1 |
| **6** | **36 - Illustrative Mathematics 1020**  Practical problem solving | Using Normal Distribution: Activities on a Casio Calculator  <http://mei.org.uk/files/ict/casio-statistics-tasks.pdf#page=6>  Worth exploring the different ways of finding probabilities given.  Use the calculator to complete Exercise 8C from the “CIMT stats\_ch8 Normal” worksheet <http://www.cimt.org.uk/projects/mepres/alevel/stats_ch8.pdf> | As above | Worksheet 1: <https://2017.integralmaths.org/pluginfile.php/37894/mod_resource/content/0/meia2sn1ax_level1.pdf> |
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| **7** | The height of trees is Normally distributed N(15, 6).  What height are 50% trees taller than?  What height are 17% taller than?  2.5% taller than? | Finding values from probabilities – the inverse normal distribution  Using Inverse Normal Distribution: Activities on a Casio Calculator  <http://mei.org.uk/files/ict/casio-statistics-tasks.pdf#page=6>  Worth exploring the different ways of finding probabilities given.  Use the calculator to complete Exercise 8D from the “CIMT stats\_ch8 Normal” worksheet <http://www.cimt.org.uk/projects/mepres/alevel/stats_ch8.pdf> | Make sure you are confident using the Inverse Normal function Also make sure you can manipulate the data when a probability less than 0.5 is given in the question. | **36 - Normal distribution questions**  **Section 2** |
| **8** | **36 - Illustrative Mathematics 1218**:  Applying methods to answer question and reporting answer | Problem solving with matching cards  **36 - Chapter 6 Normal Probability Cards 1314**  Example: finding missing mean and standard deviation: Forming and solving equations. |  | Worksheet 2 <https://2017.integralmaths.org/pluginfile.php/37895/mod_resource/content/0/meia2sn1ax_level2.pdf>  **MSV-18 the Coffee Problem (really stretching task for best students)**  [**http://www.s253053503.websitehome.co.uk/msv/msv-18.html**](http://www.s253053503.websitehome.co.uk/msv/msv-18.html)  **36 - Normal Distribution questions**  Section 3 |
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| **9** | Kirsty is measuring the length of time people wait for busses at the stop outside her school. She expects it to be normal because it is continuous and so can’t be binomial. What do you think? | When to use the normal and binomial distributions, and when not to. Other distributions available beyond A level maths.  Key facts: Normal is continuous, symmetrical, bell shaped.  Binomial is discrete, independent trial, fixed probability, fixed number of trials.  **Choosing Distribution Activity**  **(To be made) Past exam questions with distribution blacked out or missing.** |  | MEI Section Test multiple Choice  <https://integralmaths.org/pluginfile.php/11417/mod_resource/content/0/s2n1q.pdf>  MEI Online Section Test  <https://integralmaths.org/mod/quiz/attempt.php?attempt=528529>  **36 - Normal distribution questions**  Section 4 |
| **10** | **Find p(x>12) for B(200, 0.7).**  **Why is this difficult?** | **Binomial approximation**  **If we approximate one distribution with another, what do we want to match?**  **Investigating a normal approximation to the binomial**  [**https://www.geogebra.org/m/gf69VgxA**](https://www.geogebra.org/m/gf69VgxA) |  |  |
| **11** |  | **Continuity correction**  **CC matching activity**  Use the calculator to complete Exercise Exercise 8E (leave out questions using other distributions)  from the “CIMT stats\_ch8 Normal” worksheet <http://www.cimt.org.uk/projects/mepres/alevel/stats_ch8.pdf> |  |  |
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| **12** |  | **Consolidation**  Use the calculator to complete Exercise 8.7 Misc exercise (leave out questions using other distributions)from the “CIMT stats\_ch8 Normal” worksheet <http://www.cimt.org.uk/projects/mepres/alevel/stats_ch8.pdf> |  |  |