**Y13 A level Mathematics**

**22 Proof 1 week**

## Teaching objectives

**a** Understand and use proof by contradiction

**b** Recall proofs of the irrationality of √2 and the infinity of primes

**c** Apply the method in unfamiliar contexts

**Resources for advance preparation:**

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|  | **Starter** | **Main teaching**Including key questions, key teaching points, models and resources | **Notes**Including Support and Extension | **Consolidation/Plenary**Including key questions and homework |
| 1 | [**MEI: Proofs of the Irrationality of √2**](http://mei.org.uk/files/sow/21-proof-res.pdf)This activity is useful and appropriate whether or not students have seen this in KS4. Use the card sort as the starter… | …and the discussion and extension (adapting the proof for √3 and identifying where it falls down if you try to use it for √4) as the main activity. | Looking at what happens if you try √3 and √4 helps students to secure the method. | Show that 50/25 ≈ 49/25 so that 7/5 is a good approximation to √2.**Homework:**Use a spreadsheet to investigate when 2*n*2 is close to a perfect square and thus find good rational approximations to √2. Can they use a similar method to find rational approximations to e.g. √3 and 3√10? |
| 2 | [**UM: Fundamental Theorem of Arithmetic**](https://undergroundmathematics.org/divisibility-and-induction/the-fundamental-theorem-of-arithmetic) | Play [**UM: Division Game**](https://undergroundmathematics.org/divisibility-and-induction/division-game) and discuss the implications.Depending on the group, demonstrate or set the students a challenge to prove, by contradiction, that there are infinitely many prime numbers. | This is a good opportunity for developing students’ understanding of prime factorisation and some basic Number Theory, particularly useful for those students who will be taking exams such as the MAT, TMUA or STEP | **Homework:** Appropriate exercise from new textbooks, or **Integral Exercise level 1** |
| 3 | Challenge students to find the error in this [**Proof that 2 = 1**](https://www.education.com/activity/article/Algebra_Sleuth_Proof_that_1_2/) | [**UM: Why is at most one of these numbers rational?**](https://undergroundmathematics.org/thinking-about-numbers/r8276) This has a number of proofs by contradiction for students to attempt. | Students need to be able to apply the method themselves in new situations, not just be able to reproduce proofs that they have learned. | **Homework:** Appropriate exercise from new textbooks, or **Integral Exercise level 2/3** |