**Y12 AS Mathematics**

**10/21 Differentiation and Variable Acceleration 3 weeks**

## Teaching objectives

**a Understand and use the derivative of f(x) as the gradient of the tangent**

**b Use the second derivative as rate of change of gradient**

**c Differentiate xn, for rational values of n**

**d Identify where functions are increasing or decreasing**

**e Prove differentiation from first principles**

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|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **1** | [A tangent is…](https://undergroundmathematics.org/introducing-calculus/a-tangent-is)  Activity that allows students to recap their understanding of what a tangent is - can be done either interactively, or with printed resources. | [Zooming in activity](https://undergroundmathematics.org/introducing-calculus/zooming-in)  Students work in pairs on graph paper to explore what happens when they zoom into different points on the graph of y = x²  Alternatively you can use [Risp 36](https://www.stem.org.uk/system/files/elibrary-resources/2016/01/First%20steps%20into%20differentiation%20%28worksheet%29.pdf) with any graphing package.  Class discussion about what they have noticed plus [zooming in - what you might have noticed](https://undergroundmathematics.org/introducing-calculus/zooming-in/things-you-might-have-noticed) | Pair strong with weak, and be prepared to support weaker pairings.  Both activities can be done using Geogebra/Desmos, or using printed resources/graph paper. | Do a similar activity for a different graph (e.g. 2x2, 3x2, x4) - every student could have a different curve - differentiated according to strengths shown in the lesson. |
| **2** | Discuss findings from homework, compare and contrast the different curves from different students. | [Gradient spotting activity](https://undergroundmathematics.org/calculus-of-powers/gradient-spotting) - requires online access.  Teacher to demonstrate by exploring the changing gradient function using the geogebra interactive activity for y=x2.  Then students explore different quadratics to try and identify how the gradient function is affected by changing a, b and c. What patterns can they spot, and can they say why these are occurring?  Class discussion to tie this together.  Students then explore other powers (x3, x4, x), and try to spot the pattern for each of these gradient functions. | Students who spot the patterns quickly could start thinking about other powers (x3, x4, x), and how the gradient functions for each of these looks.  Those who have already looked at this can investigate non-unitary coefficients for higher powers. | To explicitly define the gradient function for a given set of functions, using the online resource. |
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| **3** | Recap finding the equation of a line between two points. | Project a graph of y=x2.  Find the equation of the line through the points with coordinates:  (3, 9) and (3.5, 12.25)? (3,9) and (3.1,9.61)? (3,9) and (3.01,9.0601)?  Students then repeat this for another point of their choosing (e.g. 2.5, 2.1, 2.01). Make sure as many different points as possible are chosen.  Class discussion to find that all points end up with y=2x.  Repeat the process as a whole class using x, x+h to derive differentiation formula.  Use the ‘new’ formula to compare with answers from the previous lesson’s homework, and identify that this works for different initial functions. | Those working more quickly can work with even finer intervals (3.001 etc.) | Worksheet/Exercise to gain routine practice from old resources or integral. |
| **4** | Students recap understanding from GCSE that gradients of distance-time graphs give speed, and gradients of speed-time graphs give acceleration - use old GCSE Qs, such as parts of questions found [here.](http://www.mathsgenie.co.uk/resources/velocitytimegraphs.pdf) | Using calculus to find speed from displacement/time, and acceleration from velocity/time graphs.  [walk-sorting activity](https://undergroundmathematics.org/introducing-calculus/walk-sorting)  Students use this activity to match up displacement-time and velocity-time graphs (assuming that students know the difference between distance and displacement - if not then [this activity](https://undergroundmathematics.org/introducing-calculus/discussing-distance))  Students then work on [this activity (integral login required)](http://mei.org.uk/files/sow/20-variable-acceleration-res.pdf), using their understanding of the relationships between displacement/velocity/acceleration, to generate graphs of the other two given one. | Some students may be able to work back from the later questions. | Complete the sketched graphs. |
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| **5** | Recap the solutions from the homework. | Ensure clarity around the concept that the gradient of displacement gives velocity, and gradient of velocity gives acceleration.  Link this to the previous work of finding gradient functions (lesson 2), and identify that velocity is a gradient function of displacement, therefore to find velocity, we can differentiate displacement.  Rates of change.  Demonstrate example(s) of being given a function for displacement and finding a function for velocity and acceleration.  Students then attempt a similar exercise, purely just using their differentiation skills, but under a new context. | Support may be needed to understand that differentiating with *t* is no different to *x* | Worksheet/Exercise to gain practice working with different variables (simple as using letters other than x & t) |
| **6** | Recap solutions from homework - Up to 3 starter questions to check understanding | Students undergo their own [experiment](https://www.stem.org.uk/resources/elibrary/resource/31129/runaway-train) to appreciate and understand the reality of how a function of x relating to t works.  Students think more in depth about modelling, including assumptions. | Some students may need a reminder on the general shape of functions.  Some students may be able to predict what type of function we will end up with. | Students to list all possible assumptions of the experiment.  Students to use their knowledge from previous lessons to find a function for velocity and acceleration for the train. |
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| **7** | Review train experiment, especially the process of differentiating displacement to find velocity and acceleration functions | Provide two or more displacement-time graphs (they could be with or without context), such that they are nonlinear, and you have at least one with a maximum and one with a minimum.  If no context, the class can discuss to all agree on what is happening in these graphs.  Students then (possibly after another demonstration) produce functions and graphs (using calculator or software) for the velocity and acceleration of each graph.  Paired/grouped discussion around what is the same/different about various points in time on each set of graphs.  Class to then make a start on [this activity](https://undergroundmathematics.org/calculus-meets-functions/gradients-of-gradients/download/index.pdf). ([Main page](https://undergroundmathematics.org/calculus-meets-functions/gradients-of-gradients/noticed)) | Provide as much or as little guidance, dependent on your class, and how you want them to take on the next activity. | Continue second investigation. |
| **8** | Review second task, identifying that:   * whenever the first differential is 0 we have a stationary point * The same point in time on the second differential tells us what type of stationary point it is | Use this new information to review the first task from previous lesson, allowing students to check for themselves that these rules work.  Go through one new example as a class, with student questioning (machine gun modelling).  Then demonstrate that the same information can be found with no graphs, and just the function - point out whenever possible the similarities and links to what has already been done over the last two lessons.  Students attempt similar problem on whiteboards (or similar), review as whole class.  Students then begin working on function-only based problems independently. | Students who have quickly understood can help with this too. | Complete set of problems as homework. |
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| **9** | Check work from end of last lesson/homework, going through problems as a whole class where necessary. | Use a variety of problems of context (some can be found [here](http://pmt.physicsandmathstutor.com/download/Maths/A-level/C2/Topic-Qs/Edexcel-Set-1/C2%20%20Differentiation%20-%20Stationary%20points.pdf)), that require students to use their understanding of double differentiation to solve more practical problems regarding rates of change.  Go through one or two that are teacher-led.  Students can then work on one question individually/paired/grouped in depth, and present their entire thought process to someone else. | Depending on class size/skill, it may be that the presenting is kept to smaller groups/pairs.  Difficulty of questions could be targeted dependent on students. | Students to complete similar questions independently. |