**Y12 AS Mathematics**

**Trigonometry 2 weeks**

## Teaching objectives

**a** To review knowledge of trigonometry from GCSE.

**b** To ensure students understand and have efficient routines for finding missing sides and angles in right angled triangles and can apply them in problem solving situations.

**c** To understand, sketch and use the graphs of the sine, cosine and tangent functions as well as transformations of these.

**d** To solve simple trigonometric equations, finding all the solutions in a given range (including quadratic equations in sin, cos tan and equations involving multiples of the unknown angle)

**d** To understand use the identities tanx=sinx/cosx and sin^2x+cos^2x=1, including using them to help solve trigonometric equations.

**e** To ensure students understand and have efficient routines for finding missing sides and angles in right angled and non-right angled triangles.

**f** To understand and know efficient routines for finding the area of a triangle (with or without the perpendicular height).

**g** To gain fluency and develop problem solving skills in the context of trigonometry

Notes:

Lesson 1 could be done entirely via starters and HW

Lessons 3,4,5 could be squeezed into two lessons if time is tight

A strong group with good recall from GCSE could do lessons 7 & 8 in one

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|  | **Starter** | **Main teaching**  Including key questions, key teaching points, models and resources | **Notes**  Including Support and Extension | **Consolidation/Plenary**  Including key questions and homework |
| **1** | Students to produce mind map or similar detailing all their remembered knowledge of trigonometry. Share in pairs/ groups, bring together key points as a class.  To be included: formulae, diagrams, graphs.  This will be revisited at the end of the unit. | SOHCAHTOA problem solving questions.  Eg. By drawing an equilateral triangle find the exact values of sin30, cos30 and tan30 and for 60.  By drawing a right angled triangle find the exact values of sin45, cos45, tan45  [UM: how high am I?](https://undergroundmathematics.org/trigonometry-triangles-to-functions/how-high-am-i/suggestion) | Extension Qs:  [MEI C4 paper January 2010 Q6i)](http://mei.org.uk/files/papers/c410ja_4754.pdf)  MEI C4 paper June 2016 Q5 (in part 1 change sec^3(θ)=2 to cos^3(θ)=1/2)  [Nrich Q](https://nrich.maths.org/2357) | Students share solutions  **Homework:** Routine SOHCAHTOA practice from old resources |
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| **2** | Use the SOHCAHTOA definitions to question how we define sin100 etc.  Use [UM: from stars to waves](https://undergroundmathematics.org/trigonometry-triangles-to-functions/from-stars-to-waves) to show the motivation from the unit circle (if you have more teaching time you could include the earlier parts detailing the origin of the sine function). Some questions for students to answer within this. | [Standards unit lesson on exploring trig graphs](http://www.mrbartonmaths.com/resources/standard%20unit%20pdfs/SU%20Algebra%20Lessons/A12%20-%20Exploring%20Trigonometrical%20Graphs.pdf) (assuming transformations already covered in sow – this lesson could be used when these are covered instead) | Detail in standards unit. Mixed ability groups or pairs.  Geogebra used in UM link. | **Homework:**  [Graph matching activity](https://nrich.maths.org/6481) (give students link to avoid colour printing)  or routine practice from old resources/ textbook |
| **3** | Give out accurate sine, cosine and tangent graphs from  --360 to 720 degrees. Students to find 4 angles with the same a) sine function. What relationships can you find between your answers? How could you find angles with the same sine function but not shown on the graph? Discuss in groups. Repeat for: b) cosine function c) tangent function. | Introduce trig equations and use of the arc functions  Find one angle x for which sinx=0.3, now find 2 more. Find all the angles between -360 and 360  Repeat for sinx= -0.3  Summarise the techniques you used. Do you need the graphs or could you do it without?  Repeat for cosx=+/-0.3  Tan x= +/-0.3  Bring methods together and generalise (if you like CAST you could introduce it here)  Mini whiteboards solving trig equations like 2tanx+1=4  for a given range. | Pair strong with middle, middle with weak and be prepared to support any of the middle/weak pairings.  Encourage stronger students to generalise – how can you efficiently find all other required angles with the same given function? | **Qs taken from MEI SOW units**  **(**[**http://mei.org.uk/2017-sow**](http://mei.org.uk/2017-sow)**)**   * Make up three trigonometric equations to solve that show you understand the symmetry of the three trigonometric curves. * Tell me the property that  and  must have in order that   **Homework:**  Fluency practice of solving simple trig equations from old resources/ textbook |
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| **4** | Starter:  Quadratic eqns related to trig ones used later– see attachment ‘Quadratic trig equations worksheet’ | Match each trig question with a similar quadratic question from starter. Then solve.  Equation set 3 from:  <http://mei.org.uk/files/sow/05-trigonometry-res.pdf> | Last 2 questions in the starter are more challenging due to inclusion of x & y. Stronger students can be encouraged to use correct mathematical notation in their solutions. | Plenary or extension Q  <https://undergroundmathematics.org/trigonometry-triangles-to-functions/r8136>  **Homework:**  Routine practice from old resources/ textbook |
| **5** | Mini-whiteboards:  Practice solving trig equations | Extend to multiple angles. Deliberate error task - card sort. (see attachment – ‘Trig equations with multiple angles error card sort’)  Students to identify correct cards and order them, and to describe the mistakes made in the other cards.  Tarsia activity:  <https://www.tes.com/teaching-resource/trig-equations-tarsia-6395151> | Ask stronger students to generalise- How can you always avoid making mistakes when the subject of the function is more complex than x? Feed back methods to class | HW:  Routine practice from old resources/ textbook |
| **6** | <https://undergroundmathematics.org/trigonometry-triangles-to-functions/slices-of-pi> | Starter should prompt the need to find out where sinx=cosx, tanx=sinx, tanx=cosx. Discuss ideas of how to do this and bring out need for algebraic solution.  Students draw right angled triangle, label and write down SOHCAHTOA rules. Ask them to work out sinx/cosx and sin^2x+cos^2x from those rules to discover identities. Apply to solving above problems.  <http://mei.org.uk/files/sow/05-trigonometry-res.pdf> (Equation set 3 may have been used earlier) | Pair strong with middle, middle with weak and be prepared to support any of the middle/weak pairings.  Ask stronger students for how to apply the trig identities to the problems. For tanx=cox how can we avoid losing the 0 solution? | Plenary or extension Q:  <https://undergroundmathematics.org/trigonometry-triangles-to-functions/r5269>  HW  Research proof of trig identities for angles larger than 90  Or fluency practice |
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| **7**  **(&8)** | Students present proofs of trig identities for angles larger than 90 (or later if more time needed for prep)  If using 1 lesson, go straight into proof of sine rule task. | Proof of sine rule – individual or pair work  <https://undergroundmathematics.org/trigonometry-triangles-to-functions/sine-ing-the-way/problem>  Recap of cosine rule and area of a triangle.  Problem solving questions sourced from old exam papers. Eg from <http://www.physicsandmathstutor.com/a-level-maths-papers/c2-by-topic/> (adjustment may be needed for questions in radians) | [Nrich link for proving cosine rule](https://nrich.maths.org/7787) (you could direct them here to help with HW)  Ability groupings can be used, with all questions needing to be completed by each group, then time given for each student to explain their hardest question to others in their group.  If using 2 lessons more time can be spent on sharing ideas and solutions in groups/ as a class. | * How would you explain why there are two triangles  with the properties ?   HW  Research proof of cosine rule and area of a triangle (especially if 2 lessons used, at beginning of 2nd lesson students can present their proofs)  Or fluency practice – including ambiguous case |
| **9** | Students to revisit earlier mind maps and add new knowledge. | Review lesson with mix of problem solving/ exam style questions.  Eg. <http://mei.org.uk/files/pdf/ps/pse20.pdf> includes knowledge of circles  Some old style exam questions (needs adapting to remove radians): <https://www.tes.com/teaching-resource/mixed-exercise-on-trigonometric-equations-6152805> | Mini plenaries where stronger students present how they have started harder problem could help support weaker ones. Alternatively pair or group students by ability. | Discuss key questions and approaches  **Homework:**  Topic assessment |

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|  | We need to use the inverse sine function (or arcsine) so: |
| Since | Rearranging gives |
| Since we are using the sine function we can find other solutions by:   * ±180 – any solution * Any solution ± 360 | Or  All other solutions will be out of range |
| Or  Or  Or  Or  Or  Or  All other solutions will be out of range | Rearranging gives |
|  |  |
| Solve  For | Rearranging gives |
|  | Or  Or  All other solutions will be out of range |

Solving more complex trigonometric equations

Find all the (real) solutions to these equations:



Match each of these trigonometric equations to a related equation above. In each case, use the solutions you found for and earlier to find all the solutions for in the range (to 1d.p if not exact)

Answers

4. or
5. or
6. or
7. or
8. or

11. No solutions